Multipath Channels

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1. Definition of a Multipath Channel

A typical radio link multipath channel is shown in Fig. 1.1 (copied from Proakis 2002). Here the direct path is the line of sight path, whereas there is a secondary path which is the reflected path from the ground. A second example is the plane to plane communication, where again two paths exist, one direct (line of sight) path and the other ground reflected (Fig. 1.2 (copied from Proakis 2002)).

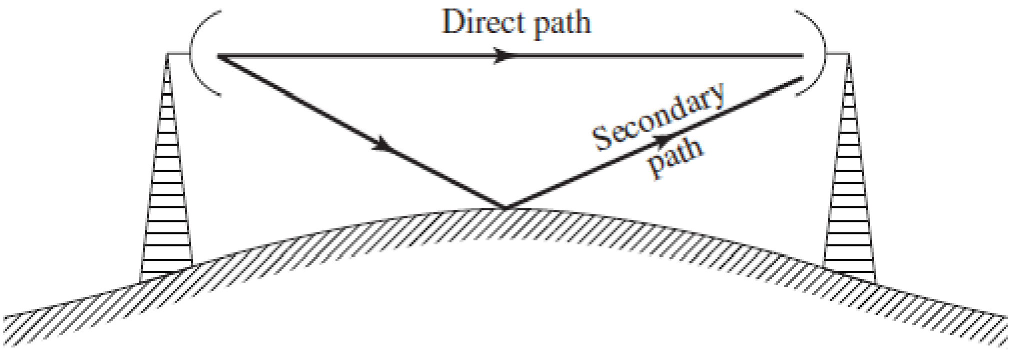

Fig. 1.1 An example of multipath (two paths) radio link communication.

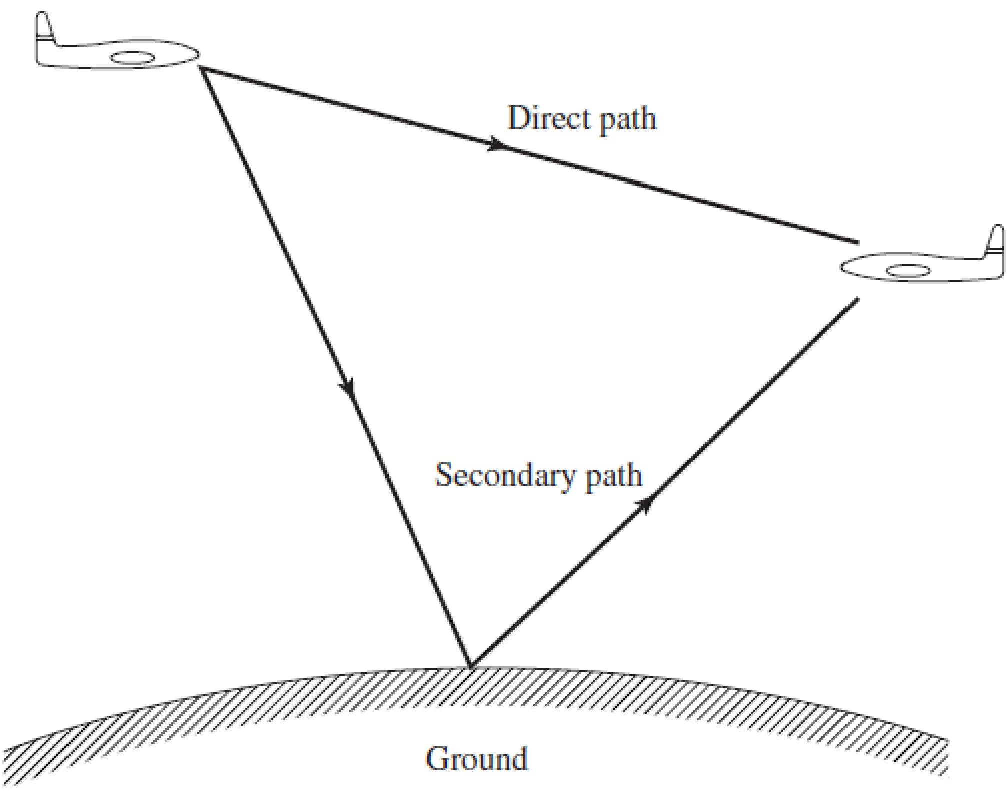

Fig. 1.2 Another example of multipath (two paths) communication between two airplanes.

For Fig. 1.1, if the medium between the two radio antennas is stable, i.e. there are no time moving objects, then we have a multipath channel response which is time invariant. But in Fig. 1.2, it is clear that we will have a time varying multipath channel since the airplanes are moving.

In general we view the time varying response of a multipath channel as shown in Fig. 1.3 (copied from Proakis 2002)

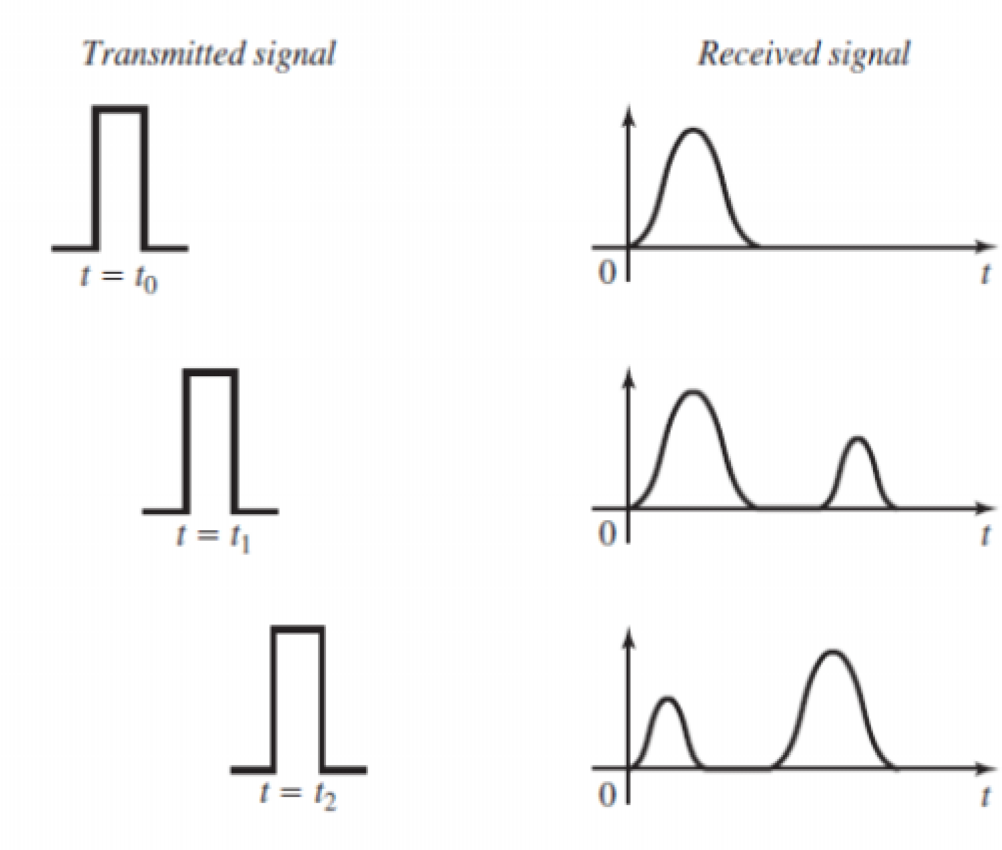

Fig. 1.3 Time response of a time varying channel.

As seen from Fig. 1.3, at time $t = t_0$, the channel acts as one path channel and imposes on low pass filtering on the transmitted signal, so we get rounding of rapid changes in the rising and falling edges. On the other hand at $t = t_1$, the characteristics of the channel has changed and the channel has become a two path channel, where the shorter path (i.e. the earlier received pulse) has almost no attenuation. Finally at $t = t_2$, the opposite of the response of $t = t_1$ occurs, such that the shorter path attenuates the signal more. It is clear from Fig. 1.3 that the channel will give a different response each time we apply an excitation (input signal). This is caused by the fact that the response of the channel is varying with time. Illustration of the transmitted and received pulses of Fig. 1. 3 along a single and continuous time axis t is given in Fig. 1.4. According to Fig. 1.4, it is better to create two time variables. One is the general time axis indicating the successive flow of message symbols from time minus infinity to plus infinity, this will be denoted by t. The other to be denoted by τ will be used to show the individual path delays. Presently, it is sufficient to consider the transmitted message signal confined to τ axis. Note that in Fig. 1.4, we have assumed zero time delay for the transmitted pulses to reach the receiver via the first (direct) path (a property that has no effect on our analysis), additionally we have removed the low pass filtering action of the channel (since channel response will be written in terms of time delta functions) and tilted the late arriving pulses. Tilting is related to the phase differences as explained below. An alternative representation in terms of the channel time response denoted by $c(t,\tau)$ is depicted in Fig. 1.5, where the axis of t and τ are separated. So each response of the input pulse is shown on an individual τ axis perpendicular to the t axis. From Fig. 1.5, we deduce that $c(t,\tau)$ can have the following form

$$c(\tau,t)=\sum_{n}\alpha_{n}(t)\exp[-j\theta_{n}(t)]\delta[\tau-\tau_{n}(t)] \tag{1.1}$$

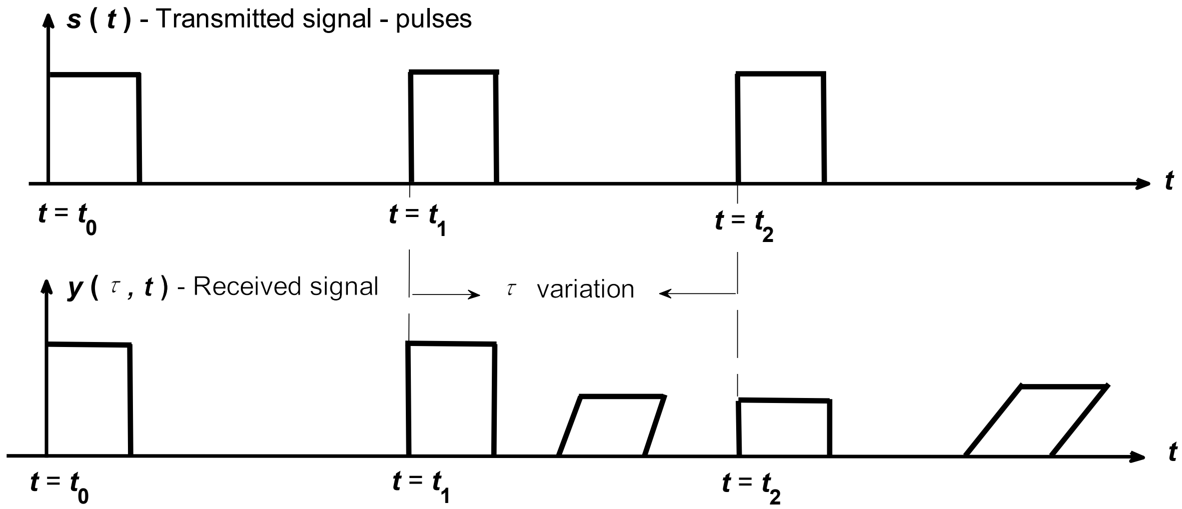

Fig 1.4 Representation of received signal for time varying channel along a single axis.

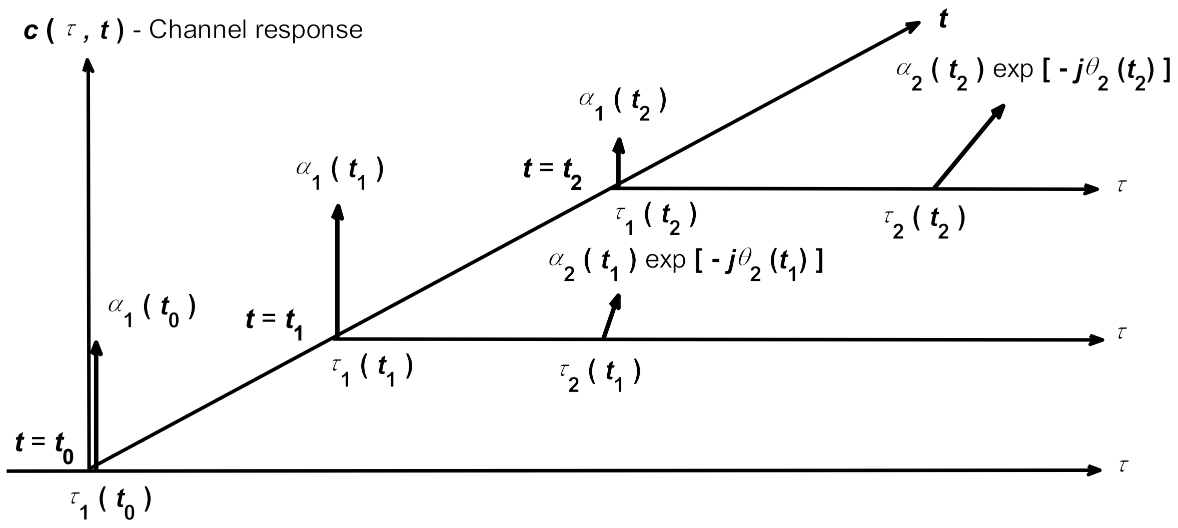

Fig 1.5 Channel response for time varying channel along two separate axes.

In (1.1), the summation over n refers to the number or paths at each t value. Hence in the case of Figs. 1.4 and 1.5, $n=1,\ 2,\ 2$ at $t=t_0,\ t_1,\ t_2$. $\alpha_n(t)$ are the attenuation coefficients distributed over the interval of one symbol to the next. In our case $\alpha_n(t)$ determine the amplitudes of delta functions , $\delta[\tau-\tau_n(t)]$. $\theta_n(t)$ is the phase (excluding the phase changes due to propagation from transmitter to receiver) in the individual $\alpha_n(t)$ components, which arise from reflections, refractions and angle of arrivals etc. For convenience we set the phase on the first (direct) path to zero, that is $\theta_1(t)=0$. Thus we show the variations in phase (of the other paths) by tilting around the horizontal axis. Finally, the time responses of the different paths are expressed by the delta functions $\delta[\tau-\tau_n(t)]$ with summation over n. Here the delta function representation is chosen for convenience. If we denote the general time variation in the frequency response of the channel by t, then the frequency response (due to τ) will be

$$\begin{aligned} C(f,t) &= \int_{-\infty}^{\infty} c(\tau,t)\exp(-j2\pi f\tau)d\tau \\ &= \int_{-\infty}^{\infty} \sum_n \alpha_n(t)\exp[-j\theta_n(t)]\delta[\tau-\tau_n(t)]\exp(-j2\pi f\tau)d\tau \\ &= \sum_n \alpha_n(t)\exp[-j\theta_n(t)]\exp[-j2\pi f\tau_n(t)] \end{aligned} \tag{1.2}$$

The last line in (1.2) reveals that the channel response contains two separate phase terms. The first one, $\theta_n(t)$ is frequency independent (but t dependent) and affects or tilts the response in its entirety. The second phase, $2\pi f\tau_n(t)$ is linearly dependent on f and merely represents the individual time delay of the paths (with respect to fastest or direct path). As seen from (1.2), for a given path and for a given time t, the absolute value of frequency response is flat, while the phase varies linearly with frequency. But from one path to the other, the channel changes its phase as a whole. The last line in (1.2) also indicates that, the output of a multipath channel will contain multiple frequency components against a single frequency excitation (input). This may also be verified by the following example.

Now assume that our transmitted message signal (input signal) consists of two complex sinusiods, that is

$$s(\tau)=a_1\exp(-j2\pi f_1\tau)+a_2\exp(-j2\pi f_2\tau) \tag{1.3}$$

The Fourier transform of $s(\tau)$ will be

$$S(f)=a_1\delta(f-f_1)+a_2\delta(f-f_2) \tag{1.4}$$

Using (1.1) to (1.4), the output from the channel will be in time and frequency domains

$$\begin{aligned} y(\tau,t) &= \sum_n \alpha_n(t)\Big(\exp\{j2\pi f_1[\tau-\tau_n(t)]-j\theta_n(t)\}+\exp\{j2\pi f_2[\tau-\tau_n(t)]-j\theta_n(t)\}\Big) \\ Y(f,t) &= [a_1\delta(f-f_1)+a_2\delta(f-f_2)]\sum_n \alpha_n(t)\exp[-j\theta_n(t)]\exp[-j2\pi f\tau_n(t)] \end{aligned} \tag{1.5}$$

So long as there is more than one path in our channel (i.e. if the channel is multipath), then more than two delta functions will be obtained from (1.5). By following a development similar to the one pursued when going from (1.3) to (1.5), for a general message signal of $s(\tau)$ sent from the transmitter, after passing through a channel of (1.1), the received signal will be

$$\begin{aligned} y(\tau,t) &= \sum_n \alpha_n(t)\, s\left[\tau - \tau_n(t)\right] \exp\left[-j\theta_n(t)\right] \\ Y(f,t) &= S(f) \sum_n \alpha_n(t) \exp\left[-j\theta_n(t)\right] \exp\left[-j2\pi f \tau_n(t)\right] \end{aligned} \tag{1.6}$$

2. Classification of Multipath Channels

Depending on the bandwidth (i.e. the spectrum width) of the transmitted message signal in relation to the delay spread of the multipath channel, we can classify channels. Assume that we have measured the path delays at various t instances and found the following spread

$$\mathrm{Max}(\tau_1,\ \tau_2 \cdots \tau_n) = T_m \tag{2.1}$$

Thus T_m expresses the maximum delay we have in a given multipath channel and since during this T_m time interval, the attenuation, $\alpha_n(t)$ and the phase, $\theta_n(t)$ coefficients remain fixed, the inverse of T_m will give us the coherence bandwidth of the channel, that is $B_{ch} = 1/T_m$. The coherence bandwidth of the channel, B_{ch} is that part of the channel frequency response, where the frequencies within B_{ch} remain correlated. This means the frequencies within B_{ch} suffer the same attenuation and the same phase shift though one individual path.

To estimate the bandwidth requirement of our transmitted message signal, we take a single pulse with a duration of T_s, then our signal bandwidth is approximately $W = 1/T_s$. For simplicity, we assume that our message has a rectangular spectrum within the bounds of $f = \pm W$. Depending on the relation between T_s and T_m or correspondingly W and B_{ch}, two distinct cases are possible

a) $T_s \gg T_m$ or $W \ll B_{ch}$: In this case the illustration given in Fig. 2.1 apply

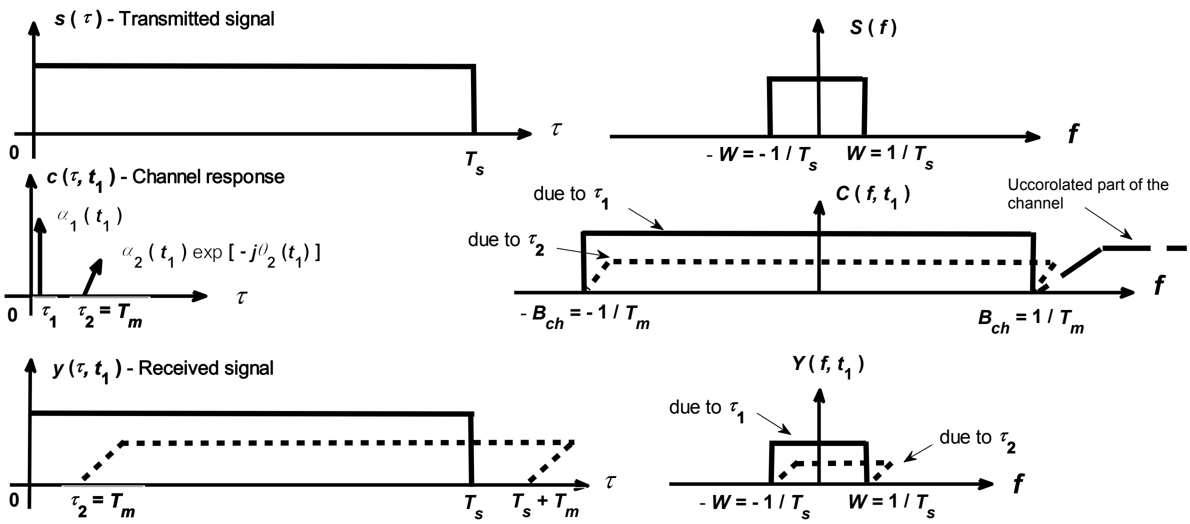

Fig 2.1 Illustration of a channel and the output when $T_s \gg T_m$.

In Fig. 2.1, we have intentionally chosen the instance of $t = t_1$ given in Figs. 1.4 and 1.5 and confined ourselves to one particular τ interval. This means that variation with respect to t is ignored for the moment. Under these circumstances, the total spectrum of the transmitted message signal passes through the channel unchanged, since the channel coherence bandwidth is much larger than the message signal bandwidth, that is $B_{ch} \gg W$. And according to (1.6), as an output, the channel will produce the multiple spectrums (in our specific case of Fig. 2.1, only two) of the transmitted message signal all overlaid within the original message signal spectrum of $|f| \leq W$. Due to variations in the phase shifts of θ_n , it may be that the receiver will have the fading of whole spectrum. This means the received signal disappears completely. Such a total collapse of the whole signal is called flat fading, sometimes called nonfrequency selective fading and will occur when $T_s \gg T_m$ or $W \ll B_{ch}$. It is clear from Fig. 2.1, a flat fading channel frequency response will have a rectangular shape, thus will not cause any amplitude distortion.

b) $T_s \ll T_m$ or $W \gg B_{ch}$: In this case the illustration given in Fig. 2.2 apply

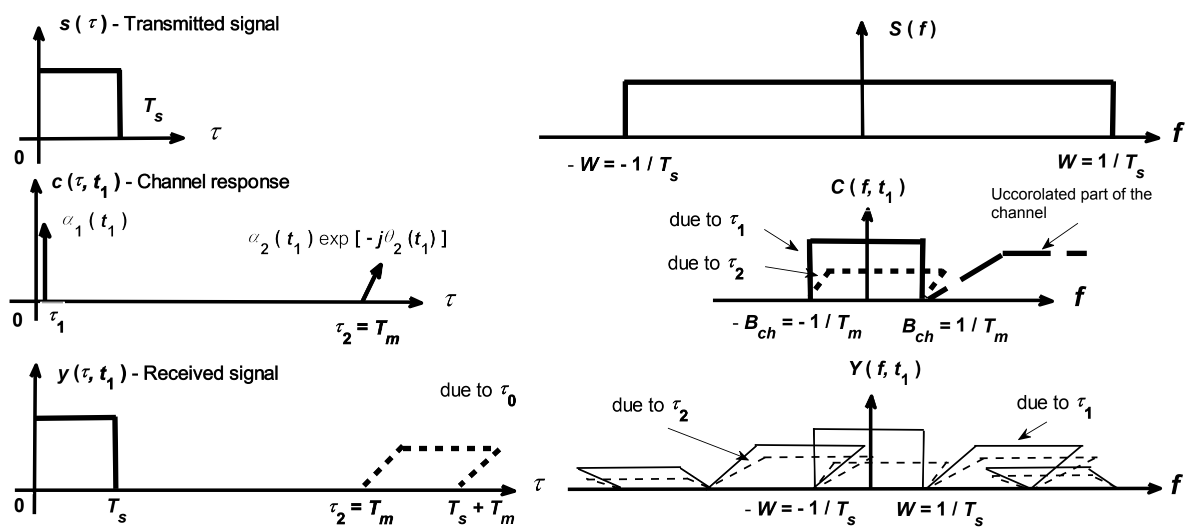

Fig 2.2 Illustration of a channel and the output when $T_s \ll T_m$.

In this case, $B_{ch} \ll W$ the channel coherence bandwidth does not cover the whole signal spectrum, thus the received signal spectrum will be divided into several frequency bands (three in Fig. 2.2) whose phases are different from each other. Note that these phase differences are again shown in Fig. 2.1 by tilts around the horizontal axis. At receiver, when the spectrum due to the secondary path, i.e. due to τ_2, is overlaid on top of the spectrum due to the first (direct) path, i.e. due to τ_1, there will be cancellations (fadings) in the spectrum, if the phase differences (due to θ_n) amount to 180^0 degrees. But now these fadings will affect only parts of the signal spectrum, thus such fadings are named frequency selective fadings. It is important to realize the response of the frequency selective channel is no longer flat along the spectrum of the message signal, thus causing amplitude distortion in the received signal.

Upon comparing Figs. 2.1 and 2.2, it is easy to deduce that signal transmission rate or signal bandwidth, i.e., $W = 1/T_s$ must be reduced if we want to avoid frequency selective fading, or correspondingly signal distortion. This is exactly what is done in orthogonal frequency division multiplexing (OFDM), where a multi carrier situation (instead of a single carrier case) is created and symbols with much extended duration then the original T_s are placed on the (orthogonal) subcarriers. This way, we convert a possible frequency selective channel into a nonfrequency selective channel.

Example 2.1 : A multipath channel has two paths, where the time delay between the two paths is 1 msec. The transmitted signal has a bandwidth of $W = 10$ kHz . Classify and describe the characteristics of this channel.

Solution : Transmitted signal bandwidth of $W = 10$ kHz means that $T_s = 1/W = 0.1$ msec . On the other hand, by setting the time delay between the two paths to delay spread $T_m = 1$ msec, we see that $T_s \ll T_m$, thus the channel is frequency selective channel, where $B_{ch} = 1/T_m = 1$ kHz. This means the signal bandwidth of 10 kHz will face frequency selective fading in slices of 1 kHz. Within 1 kHz bandwidth slices however, there will be flat fading.

Up to now, we have only considered changes along τ axis. Now we consider what happens as the t changes are also taken into account.

Although, it is easy to see form (1.6) that as t varies, then frequency shifted versions of the transmitted message signal will accumulate at the receiver. A more rigours proof is given below. For this, we go the beginning of our formulate and the output of the multipath channel as follows

$$y(t) = \int_0^\infty s(t-\tau)c(\tau,t)d\tau \tag{2.2}$$

In (2.2) we have used the variable τ to indicate time delays of paths as well as the part of the convolution integral. The reason for the lower limit of the integral starting from zero is because of causality, i.e., a transmitted signal cannot arrive at the receiver with negative delay. On the other hand, we have returned to one time variable of t at the output of the channel, since there we are at the end of the channel. The frequency response of the channel over the two time variables of τ and t involves a double Fourier transform of the channel impulse response, thus

$$C(f_\tau, f) = \int_{-\infty}^{\infty}\int_{0}^{\infty} c(\tau,t)\exp\left[-j2\pi\left(f_\tau\tau + ft\right)\right] d\tau dt \tag{2.3}$$

Note that against two time variables of τ and t, we have used two frequency variables of f_τ and f. By the Fourier transform of (2.2), also using (2.3), we get

$$Y(f) = \int_{-\infty}^{\infty} y(t)\exp(-j2\pi ft)dt = \int_{-\infty}^{\infty}\int_{0}^{\infty} s(t-\tau)c(\tau,t)\exp(-j2\pi ft)d\tau dt \tag{2.4}$$

Now inside the second integral of (2.4), we insert the inverse Fourier transform equivalence for $s(t-\tau)$, then get

$$s(t-\tau) = \int_{-\infty}^{\infty} S(f_1)\exp\left[j2\pi f_1(t-\tau)\right]df_1$$

$$\begin{aligned}
Y(f) &= \int_{-\infty}^{\infty} S(f_1)df_1 \int_{-\infty}^{\infty}\int_{0}^{\infty} c(\tau,t)\exp\left[j2\pi f_1(t-\tau)\right]\exp(-j2\pi ft)d\tau dt \\
&= \int_{-\infty}^{\infty} S(f_1)df_1 \int_{-\infty}^{\infty}\int_{0}^{\infty} c(\tau,t)\exp\left[-j2\pi(f-f_1)t\right]\exp(-j2\pi f_1 t)d\tau dt \\
&= \int_{-\infty}^{\infty} S(f_1)C(f_1, f-f_1)df_1 \quad , \quad \text{set} \quad f_\tau = f - f_1 \\
&= \int_{-\infty}^{\infty} S(f-f_\tau)C(f-f_\tau, f_\tau)df_\tau
\end{aligned} \tag{2.5}$$

As apparent from $S(f-f_\tau)$ on the last line of (2.5), in this case there will be frequency shifts in the message signal at amounts of f_τ. The rms value of f_τ is known as Doppler frequency spread of the channel and denoted by the parameter B_d. The inverse of B_d is named channel coherence time, that is $T_{ct} = 1/B_d$. Combined with the multipath spread parameter T_m, we redefine our channel classification as follows

a) If $T_m B_d < 1$, then the channel is said to be underspread,
b) If $T_m B_d > 1$, then the channel is said to be overspread.

It is important to realize that a channel may be under or over spread due to multipath spread alone or large Doppler spread alone or both. If the channel is overspread, i.e., $T_m B_d \gg 1$ or $T_m \gg 1/B_d$, then the estimation of the carrier phase is extremely difficult. But if $T_m \ll 1/B_d$, then the carrier can be estimated with good precision, enabling the phase coherent demodulation possible. Finally we say that channel fades slowly if the symbol duration is less that channel coherence time, that is $T_s < T_{ch}$, otherwise the channel is said to fade rapidly.

Example 2.2 : A multipath channel has the following characteristics,

$$T_m = 1 \text{ sec}, B_d = 0.01 \text{ Hz}, W = 5 \text{ Hz} \tag{2.6}$$

We select a symbol duration of $T_s = 10 \text{ sec}$. Based on this selection, find the followings

a) The coherence bandwidth, B_{ch} and coherence time T_{ct} of the channel.
b) Is the channel frequency or nonfrequency selective ?
c) Is the channel slowly or rapidly fading ?

Solution : We make the following calculations

a) $B_{ch} = 1/T_m = 1 \text{ Hz}, T_{ct} = 1/B_d = 100 \text{ sec}$.

Therefore

b) Since $W > B_{ch}$, then the channel is frequency selective.

c) Since $T_s < T_{ct}$, then the channel is slowly fading.

3. Rayleigh and Ricean Channels

In (1.1) the statistical distribution of $\alpha_n(t)\exp\left[-j\theta_n(t)\right]$ determines the fading model. After dropping the n summation index, we write

$$\alpha(t)\exp\left[-j\theta(t)\right] = \alpha_r(t) - j\alpha_i(t) \tag{3.1}$$

Then

$$\alpha(t) = \left[\alpha_r^2(t) + \alpha_i^2(t)\right]^{0.5} \tag{3.2}$$

Thus $\alpha(t)$ will always be positive. Assuming $\alpha_r(t)$ and $\alpha_i(t)$ are Gaussian distributed with zero mean, then the channel will be a Rayleigh fading channel and $\alpha(t)$ will have the following probability density function (pdf)

$$f(\alpha) = \begin{cases} \dfrac{\alpha}{\sigma^2}\exp\left(-\dfrac{\alpha^2}{2\sigma^2}\right) & \text{for } \alpha \geq 0 \\ 0 & \text{for } \alpha < 0 \end{cases} \tag{3.3}$$

On the other hand, $\alpha_r(t)$ and $\alpha_i(t)$ are Gaussian distributed with nonzero mean, then the pdf of $\alpha(t)$ will be Ricean (Matlab spelling is Rician)

$$f(\alpha)=\begin{cases}\frac{\alpha}{\sigma^2}\exp\left(-\frac{\alpha^2+s^2}{2\sigma^2}\right)I_0\left(\frac{\alpha s}{\sigma^2}\right) & \text{for } \alpha \geq 0 \\ 0 & \text{for } \alpha < 0\end{cases} \qquad (3.4)$$

Typically, in a channel where nearly all multipath components reach the receiver by reflections and none of them is dominant then the distribution of $\alpha(t)$ can be best described by Rayleigh pdf. This will mostly be the case for instance, in a Base Station Transceiver (BTS) to hand mobile set communication in a GSM network. On the other hand in a point to radio link communication, line of sight (direct) path will dominate, thus in such a case $\alpha(t)$ will have a Ricean type pdf.

In (3.3) and (3.4) σ^2 is the variance of $\alpha_r(t)$ or $\alpha_i(t)$, s^2 is the received power of the direct path. Plots of Rayleigh and Ricean distributions for different σ^2 and s^2 are given in Fig. 3.1.

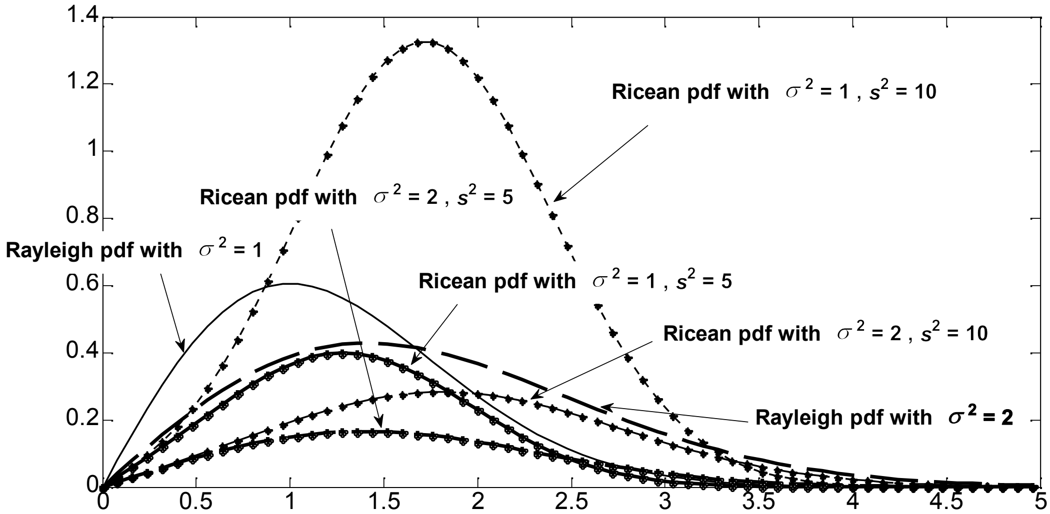

Fig. 3.1 Plots of Rayleigh and Ricean distributions for different σ^2 and s^2 .

As seen from Fig. 3.1, increases in the variance, σ^2 always spreads the pdf as expected. The increases in s^2 on the other hand cause the Ricean pdf to be more peaky. Furthermore as s^2/σ^2 is reduced, Ricean pdf turns into Rayleigh pdf, which means the disappearance of the direct component. At the other extreme however, that is $s^2/\sigma^2 \rightarrow \infty$, the direct path (line of sight) will dominate, the effect of the reflected paths will be minimal, thus we will reach an additive white Gaussian noise (AWGN) channel, where the channel response will be in the form of an unlimited channel.

Now if we consider a modulation scheme at $M=2$, where the signal waveforms are $s_1(t)=-s_2(t)$ each carrying an energy of ε_s, and transmission takes place in a multipath channel, then at receiver, the output of the matched filter, r after sampling becomes

$$r=\pm\alpha\varepsilon_s^{0.5}+n \qquad (3.5)$$

where either plus or minus sign is valid depending on whether $s_1(t)$ or $s_2(t)$ was sent from transmitter, n denotes the noise sample. Without the multipath effects, i.e. in a AWGN channel, (3.5) and the corresponding probability of error expression would be

$$r = \pm\varepsilon_s^{0.5} + n \;\;,\;\; P_e = Q\left(\sqrt{\frac{2\varepsilon_s}{N_0}}\right) \tag{3.6}$$

where Q is the complimentary error function, $N_0/2$ is the (two sided) noise spectral density. By establishing analogy between (3.5) and (3.6), we deduce that in the case of a multipath channel, for a single given α, the probability of error, $P_e(\alpha)$ will be

$$P_e(\alpha) = Q\left(\sqrt{\frac{2\alpha^2\varepsilon_s}{N_0}}\right) \tag{3.7}$$

The probability of error averaged over all α values will be obtained from

$$P_e = \int_0^\infty f(\alpha)P_e(\alpha)\,d\alpha \tag{3.8}$$

By inserting the Rayleigh pdf in (3.8) from (3.3), we find

$$P_e = 0.5\left(1 - \sqrt{\frac{\mathrm{SNR_{MP}}}{1+\mathrm{SNR_{MP}}}}\right) \;\;,\;\; \mathrm{SNR_{MP}} = 2\sigma^2\frac{\varepsilon_s}{N_0} \tag{3.9}$$

where $\mathrm{SNR_{MP}}$ may be interpreted as the (reduced) signal to noise ratio due to multipath effects of the channel. Bearing in mind that the P_e expression of an AWGN channel given in (3.6) will decay exponentially with increasing SNR (at relatively high SNR values), while the P_e of a multipath channel given in (3.9) will decay in inverse proportionality to SNR, it is easy to deduce that the performance of a multipath channel will always be inferior to AWGN channel.

Exercise 1 : On the course webpage, there are the Matlab files of MaryPe_MDPSK.m, M_DPSK_Mod.mdl, M_DPSK_deMod.mdl, ricchannelc.mdl which serve to demonstrate the P_e performances of AWGN channel, Ricean channel with three different settings of the power on the direct path and Rayleigh channel. With the present settings, the curves produced by running MaryPe_MDPSK.m confirm the above predictions, i.e. in a Ricean channel as s^2/σ^2 is increased, we obtain a P_e performance of AWGN channel. If we reduce s^2/σ^2 on the other hand, P_e performance of a Ricean channel approaches that of Rayleigh channel. By varying Kfac1, Kfac2, Kfac3, TetaLOS, PathD and AvepathG on lines 16 and 17 of MaryPe_MDPSK.m, observe and record the different P_e curves. Also observe variations against M.

Note that in this exercise, to establish a one to one correspondence to the formulation given in (3.9), the variance must be calculated. This is an exercised planned for future. Further note that in a multipath channel DPSK, which uses relative phase relations between signal vectors rather than PSK which uses absolute phase values of signal vectors, is used.