Çankaya University – ECE Department – ECE 588 (FE)

Student Name : Date : 26.05.2014
Student Number : Open source exam

Questions

1. (40 Points) At the end of a communication channel containing ISI, the pulse received is

$$x(t) = \frac{1}{1+(t/T)^4} \tag{1.1}$$

Plot $x(t)$ and read the numeric values at $t = \mp 3T,\ \mp 5T/2,\ \mp 2T,\ \mp 3T/2,\ \mp T,\ \mp T/2,\ 0$. In the equalizers to be designed, the tap spacing is arranged to be at $\tau = T/2$ and $\tau = T$, while the sampling is carried out at $t = mT$. Determine the c_n tap coefficients, i.e. column vector $\mathbf{c}$, if the number of taps is 3, 5, 7 for each case separately. Comment how much improvement is brought by the increase in number of taps.

Hint : Perform your computation in Matlab.

Solution : The plot of $x(t)$ in (1.1) is given in Fig. 1.1. Note that symmetry is taken into account when writing the data cursor values.

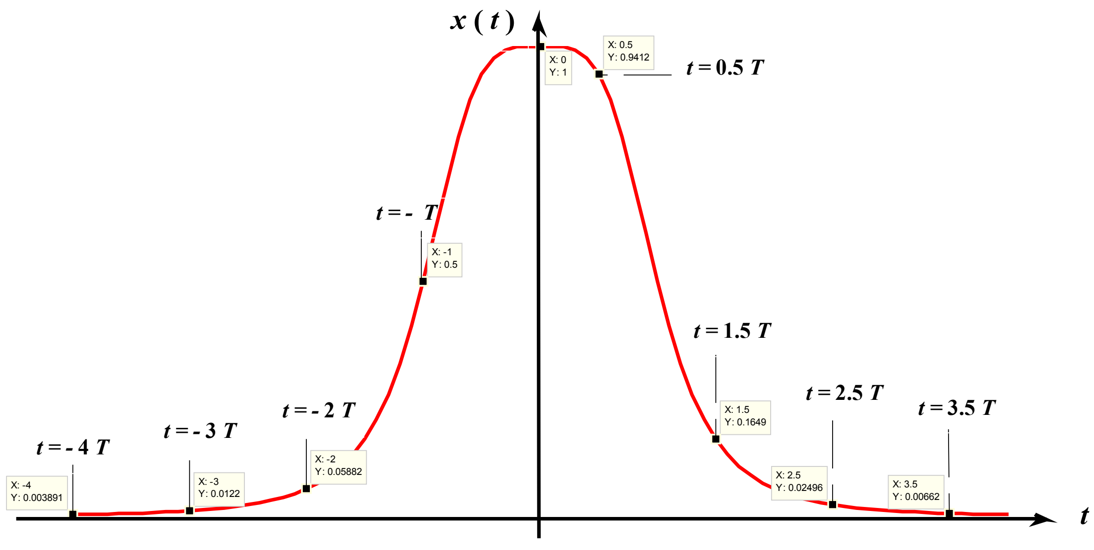

Fig. 1.1 Plot of $x(t)$ and the values at $t = \mp 3T,\ \mp 5T/2,\ \mp 2T,\ \mp 3T/2,\ \mp T,\ \mp T/2,\ 0$

The general equalizer equation is

$$q(mT) = \sum_{n=-N}^{N} c_n x(mT - n\tau) \begin{cases} 1 & m = 0 \\ 0 & m = \pm 1,\ \pm 2 \cdots,\ \pm N \end{cases} \tag{1.2}$$

In this question, $\mathbf{X}$ matrix must be constructed for the following options

a) $\tau = T/2,\ N = 1$

b) $\tau = T/2,\ N = 2$
c) $\tau = T/2,\ N = 3$
d) $\tau = T,\ N = 1$
e) $\tau = T,\ N = 2$
f) $\tau = T,\ N = 3$

By using (1.1) and Fig. 1.1, we have

$$\mathbf{X}_{a} = \begin{pmatrix} 0.1649 & 0.5 & 0.9412 \\ 0.9412 & 1 & 0.9412 \\ 0.9412 & 0.5 & 0.1649 \end{pmatrix} \quad \mathbf{X}_{b} = \begin{pmatrix} 0.0122 & 0.025 & 0.0588 & 0.1649 & 0.5 \\ 0.0588 & 0.1649 & 0.5 & 0.9412 & 1 \\ 0.5 & 0.9412 & 1 & 0.9412 & 0.5 \\ 1 & 0.9412 & 0.5 & 0.1649 & 0.0588 \\ 0.5 & 0.1649 & 0.0588 & 0.025 & 0.0122 \end{pmatrix} \tag{1.3}$$

$$\mathbf{X}_{c} = \begin{pmatrix} 0.0024 & 0.0039 & 0.0066 & 0.0122 & 0.025 & 0.0588 & 0.1649 \\ 0.0066 & 0.0122 & 0.025 & 0.0588 & 0.1649 & 0.5 & 0.9412 \\ 0.025 & 0.0588 & 0.1649 & 0.5 & 0.9412 & 1 & 0.9412 \\ 0.1649 & 0.5 & 0.9412 & 1 & 0.9412 & 0.5 & 0.1649 \\ 0.9412 & 1 & 0.9412 & 0.5 & 0.1649 & 0.0588 & 0.025 \\ 0.9412 & 0.5 & 0.1649 & 0.0588 & 0.025 & 0.0122 & 0.0066 \\ 0.1649 & 0.0588 & 0.025 & 0.0122 & 0.0066 & 0.0039 & 0.0024 \end{pmatrix} \tag{1.4}$$

$$\mathbf{X}_{d} = \begin{pmatrix} 0.0588 & 0.5 & 1 \\ 0.5 & 1 & 0.5 \\ 1 & 0.5 & 0.0588 \end{pmatrix} \quad \mathbf{X}_{e} = \begin{pmatrix} 0.0039 & 0.0122 & 0.0588 & 0.5 & 1 \\ 0.0122 & 0.0588 & 0.5 & 1 & 0.5 \\ 0.0588 & 0.5 & 1 & 0.5 & 0.0588 \\ 0.5 & 1 & 0.5 & 0.0588 & 0.0122 \\ 1 & 0.5 & 0.0588 & 0.0122 & 0.0039 \end{pmatrix} \tag{1.5}$$

$$\mathbf{X}_{f} = \begin{pmatrix} 0.0008 & 0.0016 & 0.0039 & 0.0122 & 0.0588 & 0.5 & 1 \\ 0.0016 & 0.0039 & 0.0122 & 0.0588 & 0.5 & 1 & 0.5 \\ 0.0039 & 0.0122 & 0.0588 & 0.5 & 1 & 0.5 & 0.0588 \\ 0.0122 & 0.0588 & 0.5 & 1 & 0.5 & 0.0588 & 0.0122 \\ 0.0588 & 0.5 & 1 & 0.5 & 0.0588 & 0.0122 & 0.0039 \\ 0.5 & 1 & 0.5 & 0.0588 & 0.0122 & 0.0039 & 0.0016 \\ 1 & 0.5 & 0.0588 & 0.0122 & 0.0039 & 0.0016 & 0.0008 \end{pmatrix} \tag{1.6}$$

Hence we get the equalizer coefficients for the different cases listed in (1.3) to (1.6) as

$$\mathbf{c}_a = \mathbf{X}_a^{-1} \overbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}^{q} = \begin{pmatrix} -3.0313 \\ 6.7059 \\ -3.0313 \end{pmatrix} \quad , \quad \mathbf{c}_b = \mathbf{X}_b^{-1} \overbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}}^{q} = \begin{pmatrix} 0.9789 \\ -6.3475 \\ 11.9694 \\ -6.3475 \\ 0.9789 \end{pmatrix} \quad , \quad \mathbf{c}_c = \mathbf{X}_c^{-1} \overbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}^{q} = \begin{pmatrix} -0.1171 \\ 1.3229 \\ -6.9443 \\ 12.7873 \\ -6.9443 \\ 1.3229 \\ -0.1171 \end{pmatrix} \qquad (1.7)$$

$$\mathbf{c}_d = \mathbf{X}_d^{-1} \overbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}^{q} = \begin{pmatrix} -0.8947 \\ 1.8947 \\ -0.8947 \end{pmatrix} \quad , \quad \mathbf{c}_e = \mathbf{X}_e^{-1} \overbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}}^{q} = \begin{pmatrix} 0.5513 \\ -1.342 \\ 2.2771 \\ -1.342 \\ 0.5513 \end{pmatrix} \quad , \quad \mathbf{c}_f = \mathbf{X}_f^{-1} \overbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}^{q} = \begin{pmatrix} -0.3331 \\ 0.7944 \\ -1.5091 \\ 2.4238 \\ -1.5091 \\ 0.7944 \\ -0.3331 \end{pmatrix} \qquad (1.8)$$

As seen from the numeric values in (1.3) to (1.8), as we increase the number of taps, then the tap coefficient of the outer edges seem to decrease rapidly, this is more so for $\tau = T/2$.

The above computation is carried out in Q1_FE_26052014.m.

2. (25 Points) 10 Mbit/sec signal is to be transmitted over a multipath channel. This channel has the following response

$$c(\tau,t)=\sum_{n}\alpha_n(t)\exp\left[-j\theta_n(t)\right]\delta\left[\tau-\tau_n(t)\right] \qquad (2.1)$$

The average values of amplitude, phase and time delay for up to 4 paths are given below

a) Path1 : $\langle\alpha_1(t)\rangle=0.6,\ \langle\theta_1(t)\rangle=0,\ \langle\tau_1(t)\rangle=0$

b) Path2 : $\langle\alpha_2(t)\rangle=0.1,\ \langle\theta_2(t)\rangle=\pi/6,\ \langle\tau_2(t)\rangle=0.15\ \mu\text{sec}$

c) Path3 : $\langle\alpha_3(t)\rangle=0.08,\ \langle\theta_3(t)\rangle=\pi/4,\ \langle\tau_3(t)\rangle=0.25\ \mu\text{sec}$

d) Path4 : $\langle\alpha_4(t)\rangle=0.04,\ \langle\theta_4(t)\rangle=\pi/3,\ \langle\tau_4(t)\rangle=0.35\ \mu\text{sec}$

Determine what Mary modulation, if any, has to be used to pass 10 Mbit/sec message signal through this channel. Classify this channel in terms of the given parameters.

Solution : The given channel seems to be more like a Ricean channel since for the line of sight path (Path1),

$$\langle\alpha_1(t)\rangle\gg\langle\alpha_i(t)\rangle\ ,\ i=2,\ 3,\ 4 \qquad (2.1)$$

On the other hand,

$$T_m=\max\left[\langle\tau_1(t)\rangle,\ \langle\tau_2(t)\rangle,\ \langle\tau_3(t)\rangle,\ \langle\tau_4(t)\rangle\right]=0.35\ \mu\text{sec} \qquad (2.2)$$

Hence

$$B_{ch}=1/T_m\approx 2.857\text{ MHz}\ ,\ T_b=0.1\ \mu\text{sec}\ ,\ W=1/T_b=10\text{ MHz} \qquad (2.3)$$

For reliable transmission, we require $T_s\gg T_m$ or $W\ll B_{ch}$. But in this case, since Path3 and 4 are so weak, we set $T_m\rightarrow 0.15\ \mu\text{sec}$ and we choose $T_s=4T_m=0.6\ \mu\text{sec}$. Then

$$T_s/T_b=6=k \qquad M=2^k=64 \qquad (2.4)$$

Hence, to pass 10 Mbit/sec through this channel, we have to employ 64 QAM, PSK etc.

3. (35 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) An equalizer is used to share the raised cosine functionality between transmitter and receiver filters : False, an equalizer attempts to equalize the channel response, $C(f)$.

b) A point to point radio channel operates as Rayleigh channel : False, it operates as Ricean channel.

c) A satellite link (uplink or downlink) operates as Ricean channel : True, since there is no multipath effects in such a channel.

d) We have to transmit at lower bit rates, as the channel becomes overspread : True, overspread channel causes frequency selective fading, to avoid this, we reduce out bit rate (or symbol rate) by converting from serial data stream into parallel symbols, place the parallel symbols onto orthogonal subcarriers, thus allocating narrow band slices to each symbol (OFDM).

e) We face ISI problem because the atmospheric channels are time varying : False, we face ISI problem due to band limitations.

f) The frequency and time responses of multipath channels vary with time : True, this is precisely the definition of time varying channel.