Çankaya University – ECE Department – ECE 588 (FE)

Student Name : Date : 25.05.2015
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Questions

1. (40 Points) At the end of a communication channel containing ISI, the pulse received is

$$x(t) = \exp\left(-t^2 / 2T^2\right) \quad (1.1)$$

Plot $x(t)$ and read the numeric values at $t = \mp 4T,\ \mp 3T,\ \mp 5T/2,\ \mp 2T,\ \mp 3T/2,\ \mp T,\ \mp T/2,\ 0$. In the equalizers to be designed, the tap spacing is arranged to be at $\tau = T/2$ and $\tau = T$, while the sampling is carried out at $t = mT$. Determine the c_n tap coefficients, i.e. column vector $\mathbf{c}$, if the number of taps is 3, 5, 7 for each case separately. Estimate the equalization ratios, thus the improvement brought by the increase in number of taps.

Hint : Perform your computation in Matlab, using Q1_FE_26052014.m available on course webpage.

Solution : The plot of $x(t)$ in (1.1) is given in Fig. 1.1. Note that symmetry is taken into account when writing the data cursor values.

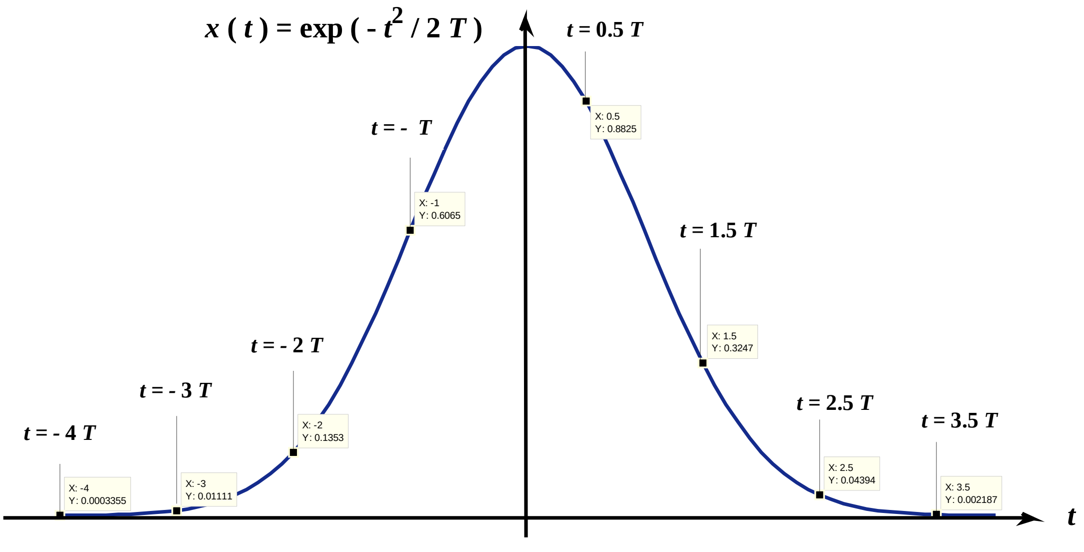

Fig. 1.1 Plot of $x(t) = \exp\left(-t^2 / 2T^2\right)$ and the values at $t = \mp 4T,\ \mp 7T/2,\ \mp 3T,\ \mp 5T/2,\ \mp 2T,\ \mp 3T/2,\ \mp T,\ \mp T/2,\ 0$

The general equalizer equation is

$$q(mT) = \sum_{n=-N}^{N} c_n x(mT - n\tau) \begin{cases} 1 & m = 0 \\ 0 & m = \pm 1,\ \pm 2 \cdots,\ \pm N \end{cases} \quad (1.2)$$

In this question, $\mathbf{X}$ matrix must be constructed for the following options

a) $\tau = T/2,\ N = 1$ (three taps, fractional spacing)

b) $\tau = T/2,\ N = 2$ (five tabs, fractional spacing)
c) $\tau = T/2,\ N = 3$ (seven taps, fractional spacing)
d) $\tau = T,\ N = 1$ (three taps, full spacing)
e) $\tau = T,\ N = 2$ (five tabs, full spacing)
f) $\tau = T,\ N = 3$ (seven taps, full spacing)

By using (1.1) and Fig. 1.1, we have

$$\mathbf{X_a} = \begin{pmatrix} 0.3247 & 0.6065 & 0.8825 \\ 0.8825 & 1 & 0.8825 \\ 0.8825 & 0.6065 & 0.3247 \end{pmatrix} \quad \mathbf{X_b} = \begin{pmatrix} 0.0111 & 0.0439 & 0.1353 & 0.3247 & 0.6065 \\ 0.1353 & 0.3247 & 0.6065 & 0.8825 & 1 \\ 0.6065 & 0.8825 & 1 & 0.8825 & 0.6065 \\ 1 & 0.8825 & 0.6065 & 0.3247 & 0.1353 \\ 0.6065 & 0.3247 & 0.1353 & 0.0439 & 0.0111 \end{pmatrix} \tag{1.3}$$

$$\mathbf{X_c} = \begin{pmatrix} 0 & 0.0003 & 0.0022 & 0.0111 & 0.0439 & 0.1353 & 0.3247 \\ 0.0022 & 0.0111 & 0.0439 & 0.0588 & 0.1649 & 0.5 & 0.9412 \\ 0.025 & 0.0588 & 0.1649 & 0.5 & 0.9412 & 1 & 0.9412 \\ 0.1649 & 0.5 & 0.9412 & 1 & 0.9412 & 0.5 & 0.1649 \\ 0.9412 & 1 & 0.9412 & 0.5 & 0.1649 & 0.0588 & 0.025 \\ 0.9412 & 0.5 & 0.1649 & 0.0588 & 0.025 & 0.0122 & 0.0066 \\ 0.1649 & 0.0588 & 0.025 & 0.0122 & 0.0066 & 0.0039 & 0.0024 \end{pmatrix} \tag{1.4}$$

$$\mathbf{X_d} = \begin{pmatrix} 0.1353 & 0.6065 & 1 \\ 0.6065 & 1 & 0.6065 \\ 1 & 0.6065 & 0.1353 \end{pmatrix} \quad \mathbf{X_e} = \begin{pmatrix} 0.0003 & 0.0011 & 0.1353 & 0.6065 & 1 \\ 0.0111 & 0.1353 & 0.6065 & 1 & 0.6065 \\ 0.1353 & 0.6065 & 1 & 0.6065 & 0.1353 \\ 0.6065 & 1 & 0.6065 & 0.1353 & 0.0111 \\ 1 & 0.6065 & 0.1353 & 0.0111 & 0.0003 \end{pmatrix} \tag{1.5}$$

$$\mathbf{X_f} = \begin{pmatrix} 0 & 0 & 0.0003 & 0.0111 & 0.1353 & 0.6065 & 1 \\ 0 & 0.0003 & 0.0111 & 0.1353 & 0.6065 & 1 & 0.6065 \\ 0.0003 & 0.0111 & 0.1353 & 0.6065 & 1 & 0.6065 & 0.1353 \\ 0.0111 & 0.1353 & 0.6065 & 1 & 0.6065 & 0.1353 & 0.0111 \\ 0.1353 & 0.6065 & 1 & 0.6065 & 0.1353 & 0.0111 & 0.0003 \\ 0.6065 & 1 & 0.6065 & 0.1353 & 0.0111 & 0.0003 & 0 \\ 1 & 0.6065 & 0.1353 & 0.0111 & 0.0003 & 0 & 0 \end{pmatrix} \tag{1.6}$$

Hence we get the equalizer coefficients for the different cases listed in (1.3) to (1.6) as

$$\mathbf{c_a} = \mathbf{X_a^{-1}} \overset{\mathbf{q}}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} = \begin{pmatrix} -4.4393 \\ 8.8354 \\ -4.4393 \end{pmatrix} \quad , \quad \mathbf{c_b} = \mathbf{X_b^{-1}} \overset{\mathbf{q}}{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} 5.9468 \\ -21.8313 \\ 32.3183 \\ -21.8313 \\ 5.9468 \end{pmatrix} \quad , \quad \mathbf{c_c} = \mathbf{X_c^{-1}} \overset{\mathbf{q}}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} -4.1075 \\ 22.0876 \\ -53.0242 \\ 70.4608 \\ -53.0242 \\ 22.0876 \\ -4.1075 \end{pmatrix} \tag{1.7}$$

$$\mathbf{c_d} = \mathbf{X_d^{-1}} \overset{\mathbf{q}}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} = \begin{pmatrix} -1.5179 \\ 2.8413 \\ -1.5179 \end{pmatrix} \quad , \quad \mathbf{c_e} = \mathbf{X_e^{-1}} \overset{\mathbf{q}}{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} 1.2314 \\ -2.9155 \\ 4.2033 \\ -2.9155 \\ 1.2314 \end{pmatrix} \quad , \quad \mathbf{c_f} = \mathbf{X_f^{-1}} \overset{\mathbf{q}}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} -0.8272 \\ 2.0877 \\ -3.6343 \\ 4.8619 \\ -3.6343 \\ 2.0877 \\ -0.8272 \end{pmatrix} \tag{1.8}$$

As seen from the numeric values in (1.3) to (1.8), as we increase the number of taps, then the tap coefficient of the outer edges seem to decrease rapidly, this is more so for $\tau = T/2$.

The sum of $x(t)$ in the range $t = -4T$ up to $t = 4T$: $E_{tq} = \sum_{t=-4T}^{4T} x(t, \tau = T)$

The sum of $x(t)$ in the range (three taps) $t = -T$ up to $t = T$: $E_{eq} = \sum_{t=-T}^{T} x(t, \tau = T)$

Equalization Ratio for three taps $R = E_{eq} / E_{eq} = 0.8829$

Equalization Ratio for five taps $R = E_{eq} / E_{eq} = 0.9909$

Equalization Ratio for seven taps $R = E_{eq} / E_{eq} = 0.9997$ (6.13)

The above computation is carried out in Q1_FE_26052014.m.

2. (25 Points) 64 Mbit/sec signal is to be transmitted over a multipath channel. This channel has the following response

$$c(\tau,t)=\sum_n \alpha_n(t)\exp\left[-j\theta_n(t)\right]\delta\left[\tau-\tau_n(t)\right] \tag{2.1}$$

The average values of amplitude, phase and time delay for up to 4 paths are given below

a) Path1 : $\langle\alpha_1(t)\rangle=0.6,\ \langle\theta_1(t)\rangle=0,\ \langle\tau_1(t)\rangle=0$

b) Path2 : $\langle\alpha_2(t)\rangle=0.1,\ \langle\theta_2(t)\rangle=\pi/6,\ \langle\tau_2(t)\rangle=0.15\ \mu\text{sec}$

c) Path3 : $\langle\alpha_3(t)\rangle=0.08,\ \langle\theta_3(t)\rangle=\pi/4,\ \langle\tau_3(t)\rangle=0.24\ \mu\text{sec}$

d) Path4 : $\langle\alpha_4(t)\rangle=0.04,\ \langle\theta_4(t)\rangle=\pi/3,\ \langle\tau_4(t)\rangle=0.05\ \mu\text{sec}$

Determine what Mary modulation, if any, has to be used to pass this message signal through the channel. Classify this channel in terms of the given parameters and write for the received signal.

Solution : The given channel seems to be more like a Ricean channel since for the line of sight path (Path1),

$$\langle\alpha_1(t)\rangle \gg \langle\alpha_i(t)\rangle\ ,\ i=2,\ 3,\ 4 \tag{2.1}$$

On the other hand,

$$T_m=\max\left[\langle\tau_1(t)\rangle,\ \langle\tau_2(t)\rangle,\ \langle\tau_3(t)\rangle,\ \langle\tau_4(t)\rangle\right]=0.24\ \mu\text{sec} \tag{2.2}$$

Hence

$$B_{ch}=1/T_m \approx 4.167\ \text{MHz}\ ,\ T_b=0.0156\ \mu\text{sec}\ ,\ W=1/T_b=64\ \text{MHz} \tag{2.3}$$

For reliable transmission, we require $T_s \gg T_m$ or $W \ll B_{ch}$. But in this case, since Path3 and 4 are so weak, we set $T_m \to 0.15\ \mu\text{sec}$ and we choose $T_s \approx T_m = 0.15\ \mu\text{sec},\ T_s/T_b \approx 10$. Then

$$T_s/T_b=k \qquad M=2^k=1024 \tag{2.4}$$

Hence, to pass 64 Mbit/sec through this channel, we have to employ 1024 QAM, PSK etc, which is a rather high Mary value, so it is best to resort to OFDM.

Since B_d is not specified, we let $B_d \to 0,\ T_{ct} \to \infty$. Hence, since $W=1/T_s \approx B_{ch}$, then the channel is at the border line of being frequency selective and nonfrequency selective fading. Furthermore, since $T_s \ll T_{ct}$, then the channel is slowly fading or nonfading.

3. (35 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) As the number of taps in an equalizer increases, equalization becomes better : True, as shown in the lecture notes entitled, “Notes on ISI_Sept 2012_HTE”.

b) A GSM base station operates as Rayleigh channel : In most case the path between the GSM base station and the mobile terminal is in the form of Rayleigh channel.

c) A satellite link (uplink or downlink) operates as multipath channel : False, since on such a path, there are no obstructions for reflections, this is dominantly one line of sight channel, or Ricean channel.

d) We can increase the transmitted bit rate, as the channel becomes more underspread : True, according to Figs. 2.1 and 2.2 of the notes entitled, “Notes on Multipath channels_Oct 2012_HTE”.

e) ISI occurs when the channel is time varying : False, ISI occurs when $1/T > 2W$ and is independent of the time variations of the channel.

f) The frequency and time responses of fibre optic cable channels vary with time : False, since fibre optic cable offers a well guided propagation medium, the channel response does not change with time.