Çankaya University – ECE Department – ECE 588 (FE)

Student Name : Date : 09.01.2017
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Questions

1. (40 Points) At the end of a communication channel containing ISI, the pulse received is

$$x(t) = \cos(2t/T)\exp(-t^2/2T^2) \quad (1.1)$$

Plot $x(t)$ and read the numeric values at
$t = \mp 4T,\ \mp 3T,\ \mp 5T/2,\ \mp 2T,\ \mp 3T/2,\ \mp T,\ \mp T/2,\ 0$. In the equalizers to be designed, the tap spacing is arranged to be at $\tau = T/2$ and $\tau = T$, while the sampling is carried out at $t = mT$. Determine the c_n tap coefficients, i.e. column vector $\mathbf{c}$, if the number of taps is 3, 5, 7 for each case separately. Estimate the equalization ratios, thus the improvement brought by the increase in number of taps.

Hint : Perform your computation in Matlab, using Q1_FE_26052015.m available on course webpage. For convenience set $T = 1$, then adjust t in increments of T. Do not forget to perform dot multiplication between the cosine and the exponential terms of (1.1).

Solution : The plot of $x(t)$ in (1.1) is given in Fig. 1.1. Note that symmetry is taken into account when writing the data cursor values.

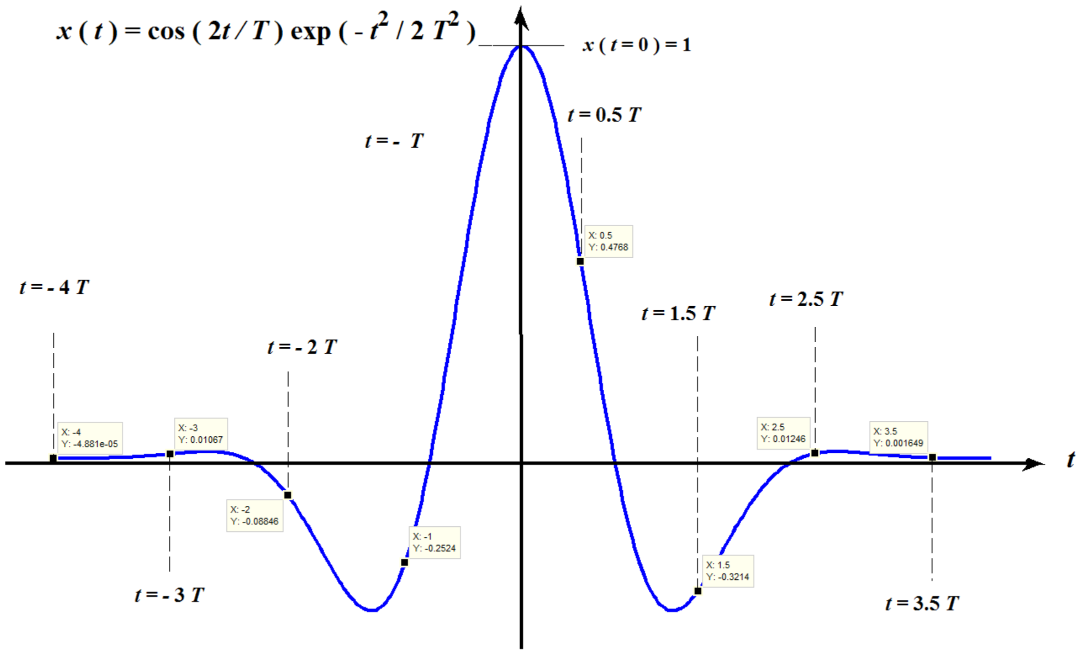

Fig. 1.1 Plot of $x(t) = \cos(2t/T)\exp(-t^2/2T^2)$ and the values at
$t = \mp 4T,\ \mp 7T/2,\ \mp 3T,\ \mp 5T/2,\ \mp 2T,\ \mp 3T/2,\ \mp T,\ \mp T/2,\ 0$

The general equalizer equation is

$$q(mT)=\sum_{n=-N}^{N} c_n x(mT-n\tau)\begin{cases}1 & m=0\\ 0 & m=\pm 1,\ \pm 2\cdots,\ \pm N\end{cases} \tag{1.2}$$

In this question, $\mathbf{X}$ matrix must be constructed for the following options

a) $\tau = T/2,\ N=1$ (three taps, fractional spacing)
b) $\tau = T/2,\ N=2$ (five tabs, fractional spacing)
c) $\tau = T/2,\ N=3$ (seven taps, fractional spacing)
d) $\tau = T,\ N=1$ (three taps, full spacing)
e) $\tau = T,\ N=2$ (five tabs, full spacing)
f) $\tau = T,\ N=3$ (seven taps, full spacing)

By using (1.1) and Fig. 1.1, we have

$$\mathbf{X}_a=\begin{pmatrix}-0.3214 & -0.2524 & 0.4768\\ 0.4768 & 1 & 0.4768\\ 0.4768 & -0.2524 & -0.3214\end{pmatrix}\quad \mathbf{X}_b=\begin{pmatrix}0.0107 & 0.0125 & -0.0885 & -0.3214 & -0.2524\\ -0.0885 & -0.3214 & -0.2524 & 0.4768 & 1\\ -0.2524 & 0.4768 & 1 & 0.4768 & -0.2524\\ 1 & 0.4768 & -0.2524 & -0.3214 & -0.0885\\ -0.2524 & -0.3214 & -0.0885 & 0.0125 & 0.0107\end{pmatrix} \tag{1.3}$$

$$\mathbf{X}_c=\begin{pmatrix}0 & 0 & 0.0016 & 0.0107 & 0.0125 & -0.0885 & -0.3214\\ 0.0016 & 0.0107 & 0.0125 & -0.0885 & -0.3214 & -0.2524 & 0.4769\\ 0.025 & -0.0885 & -0.3214 & -0.2524 & 0.4768 & 1 & 0.4768\\ -0.3214 & -0.2524 & 0.4768 & 1 & 0.4768 & -0.2524 & -0.3214\\ 0.4768 & 1 & 0.4768 & -0.2524 & -0.3214 & -0.0885 & 0.0125\\ 0.4768 & -0.2524 & -0.3214 & -0.0885 & 0.0125 & 0.0107 & 0.0016\\ -0.3214 & -0.0885 & 0.0125 & 0.0107 & 0.0016 & 0 & 0\end{pmatrix} \tag{1.4}$$

$$\mathbf{X}_d=\begin{pmatrix}-0.0885 & -0.2524 & 1\\ -0.2524 & 1 & -0.2524\\ 1 & -0.2524 & -0.0885\end{pmatrix}\quad \mathbf{X}_e=\begin{pmatrix}0 & 0.0107 & -0.0885 & -0.2524 & 1\\ 0.0107 & -0.0885 & -0.2524 & 1 & -0.2524\\ -0.0885 & -0.2425 & 1 & -0.2524 & -0.0885\\ -0.2524 & 1 & -0.2524 & -0.0885 & 0.0107\\ 1 & -0.2524 & -0.0885 & 0.0107 & 0\end{pmatrix} \tag{1.5}$$

$$\mathbf{X}_f=\begin{pmatrix}0 & 0 & 0 & 0.0107 & -0.0885 & -0.2524 & 1\\ 0 & 0 & 0.0107 & -0.0885 & -0.2524 & 1 & -0.2524\\ 0 & 0.0107 & -0.0885 & -0.2524 & 1 & -0.2524 & -0.0885\\ 0.0107 & -0.0885 & -0.2524 & 1 & -0.2524 & 0.0885 & 0.0107\\ -0.0885 & -0.2524 & 1 & -0.2524 & -0.0885 & 0.0107 & 0\\ -0.2524 & 1 & -0.2524 & -0.0885 & 0.0107 & 0 & 0\\ 1 & -0.2524 & -0.0885 & 0.0107 & 0 & 0 & 0\end{pmatrix} \tag{1.6}$$

Hence we get the equalizer coefficients for the different cases listed in (1.3) to (1.6) as

$$\mathbf{c_a} = \mathbf{X_a^{-1}} \overset{q}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} = \begin{pmatrix} 0.6372 \\ 0.3923 \\ 0.6372 \end{pmatrix} \;,\; \mathbf{c_b} = \mathbf{X_b^{-1}} \overset{q}{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} 1.464 \\ -2.2609 \\ 3.8951 \\ -2.2609 \\ 1.464 \end{pmatrix} \;,\; \mathbf{c_c} = \mathbf{X_c^{-1}} \overset{q}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} 3.8555 \\ -13.3323 \\ 23.846 \\ -25.9922 \\ 23.846 \\ -13.3323 \\ 3.8555 \end{pmatrix} \quad (1.7)$$

$$\mathbf{c_d} = \mathbf{X_d^{-1}} \overset{q}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} = \begin{pmatrix} 0.3219 \\ 1.1625 \\ 0.3219 \end{pmatrix} \;,\; \mathbf{c_e} = \mathbf{X_e^{-1}} \overset{q}{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} 0.2054 \\ 0.3969 \\ 1.2367 \\ 0.3969 \\ 0.2054 \end{pmatrix} \;,\; \mathbf{c_f} = \mathbf{X_f^{-1}} \overset{q}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} 0.0819 \\ 0.2315 \\ 0.4151 \\ 1.2488 \\ 0.4151 \\ 0.2315 \\ 0.0819 \end{pmatrix} \quad (1.8)$$

As seen from the numeric values in (1.3) to (1.8), as we increase the number of taps, then the tap coefficient of the outer edges seem to decrease rapidly, this is more so for $\tau = T/2$.

The sum of $x\ \ t$ in the range $t = -4T$ up to $t = 4T$: $E_{tq} = \sum_{t=-4T}^{4T} \left| x\ \ t, \tau = T\ \right|$

The sum of $x\ \ t$ in the range (three taps) $t = -T$ up to $t = T$: $E_{eq} = \sum_{t=-T}^{T} \left| x\ \ t, \tau = T\ \right|$

Results for full tape case

Equalization Ratio for three taps $R = E_{eq} / E_{tq} = 0.8835$

Equalization Ratio for five taps $R = E_{eq} / E_{tq} = 0.9874$

Equalization Ratio for seven taps $R = E_{eq} / E_{tq} = 0.9999$ (1.9)

The above computation is carried out in Q1_FE_09012017.m.

2. (25 Points) 16 Mbit/sec signal is to be transmitted over a multipath channel. This channel has the following response

$$c(\tau,t)=\sum_n \alpha_n(t)\exp\left[-j\theta_n(t)\right]\delta\left[\tau-\tau_n(t)\right] \tag{2.1}$$

The average values of amplitude, phase and time delay for up to 4 paths are given below

a) Path1 : $\langle\alpha_1(t)\rangle=0.4,\ \langle\theta_1(t)\rangle=0,\ \langle\tau_1(t)\rangle=0$

b) Path2 : $\langle\alpha_2\ t\ \rangle=0.1,\ \langle\theta_2\ t\ \rangle=\pi/6,\ \langle\tau_2\ t\ \rangle=0.01\ \mu\text{sec}$

c) Path3 : $\langle\alpha_3(t)\rangle=0.15,\ \langle\theta_3(t)\rangle=\pi/4,\ \langle\tau_3(t)\rangle=0.02\ \mu\text{sec}$

d) Path4 : $\langle\alpha_4(t)\rangle=0.25,\ \langle\theta_4(t)\rangle=\pi/3,\ \langle\tau_4(t)\rangle=0.03\ \mu\text{sec}$

Classify this channel. Plot an example of a received waveform indicating the effect of multichannel reflections.

Determine what Mary modulation, if any, has to be used to pass this message signal through the channel. Classify this channel in terms of the given parameters and write for the received signal.

Solution : The given channel seems to be more like a Rayleigh channel since the average attenuation coefficients seem to be of the same order, i.e.,

$$\left|\langle\alpha_i(t)\rangle\right|\approx\left|\langle\alpha_m(t)\rangle\right|\ ,\ i\neq m=1,\ 2,\ 3,\ 4 \tag{2.1}$$

On the other hand,

$$T_m=\max\left[\langle\tau_1(t)\rangle,\ \langle\tau_2(t)\rangle,\ \langle\tau_3(t)\rangle,\ \langle\tau_4(t)\rangle\right]=\langle\tau_4(t)\rangle=0.03\ \mu\text{sec} \tag{2.2}$$

Hence

$$B_{ch}=1/T_m\approx 33.33\text{ MHz}\ ,\ T_b=1/16\text{ Mbit/sec}=62.5\text{ nsec}\ ,\ W=1/T_b=16\text{ MHz} \tag{2.3}$$

Since, $W<B_{ch}$ we do not require any Mary levelling, hence $M=2$ is sufficient.

3. (35 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) To eliminate ISI, we can send the message signal as raised cosine pulses at transmitter : A better expression is that the combined responses of transmitter and receiver filters should correspond to raised cosine filtering as shown in Fig. 3.2 of lecture notes entitled, “Notes on ISI_Sept 2012_HTE”.

b) A GSM radiolink connecting base stations to the core network operates as Rayleigh channel : False, since this is a point to point communication, the direct path is dominating, hence such a communication link can only be an example of Ricean channel.

c) A communication channel becomes more coherent, as the number of paths increases : False, as the number of path increases, there is a possibility of maximum path delay increasing. This will then cause a reduction of coherence bandwidth of the channel as explained, as explained in section 2 of lecture notes entitled, “Notes on Multipath channels_Oct 2012_HTE”.

d) Channel equalization can be applied if the channel is time invariant : Not necessarily, channel equalization is also applicable to time varying channels, But in such a case, tab coefficients have to be time dependent.

e) ISI occurs when the channel response is time varying : False, ISI occurs, when adjacent symbols extend in time an start to overlap with neighbouring ones, as shown in Fig. 1.5 of lecture notes entitled, “Notes on ISI_Sept 2012_HTE”.

f) All atmospheric channels are time varying channels : Because of changes in the atmospheric conditions, atmospheric channels can be slow or fast time varying channels.