Çankaya University – ECE Department – ECE 588 (FE)

Student Name : Date : 07.01.2013
Student Number : Open source exam

Questions

1. (35 Points) At the end of a communication channel containing ISI, the pulse received is as shown below

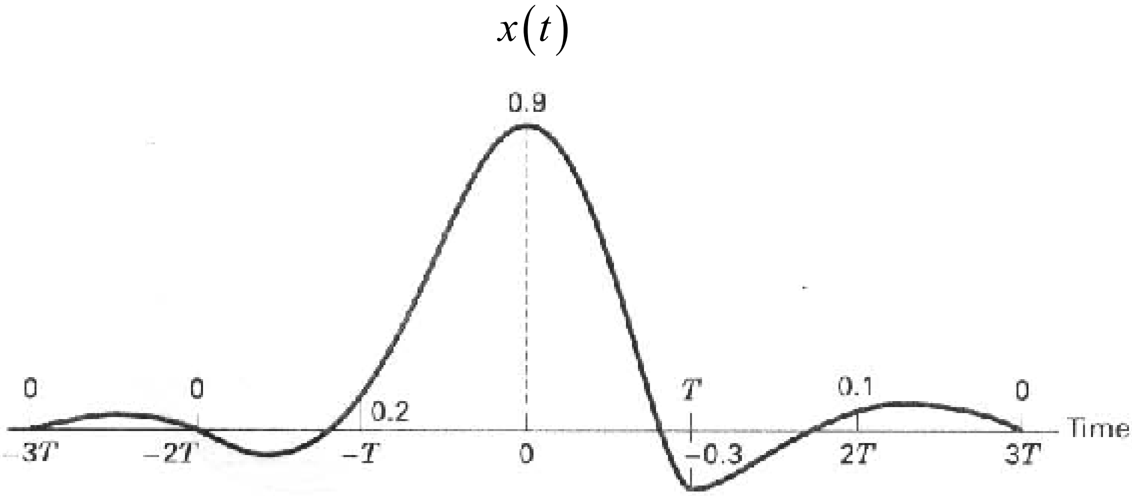

Fig. 1.1 Graph of received pulse for question 1.

From this graph, by reading the sampled values at $t = \mp 3T,\ \mp 2T,\ \mp T,\ 0$, find the c_n coefficients of zero forcing three tap and five tap equalizers operating at sampling rates of $t = \mp mT$, where m is an integer and T is symbol duration of the transmitted signal. After finding c_n coefficients, evaluate, compare and comment on $q(mT)$ values for the three tap and five tap equalizers. Assuming the transmitter and receiver filters share the raised cosine functionality equally, explain the origins of the ISI seen in the received pulse of Fig. 1. State the maximum bit rate that can be transmitted under these circumstances without ISI. State the ways of achieving higher bit rates for the same channel.

Solution : From ()6.7) of Notes on ISI_Sept 2012, we write

$$q(mT) = \sum_{n=-N}^{N} c_n x(mT - nT) \qquad (1.1)$$

To obtain a three tap equalizer we set $N = 1$ and for the five tap equalizer we set $N = 2$. We construct $\mathbf{X}$ matrix for the two cases by taking $x(mT - nT)$ and running $m = -1,\ 0,\ +1$ $n = -1,\ 0,\ +1$ for $N = 1$ and $m = -2,\ -1,\ 0,\ +1,\ 2$ $n = -2,\ -1,\ 0,\ +1,\ 2$ for $N = 2$, thus we get

$$\mathbf{X}_3 = \begin{pmatrix} 0.9 & 0.2 & 0 \\ -0.3 & 0.9 & 0.2 \\ 0.1 & -0.3 & 0.9 \end{pmatrix} \qquad \mathbf{X}_5 = \begin{pmatrix} 0.9 & 0.2 & 0 & 0 & 0 \\ -0.3 & 0.9 & 0.2 & 0 & 0 \\ 0.1 & -0.3 & 0.9 & 0.2 & 0 \\ 0 & 0.1 & -0.3 & 0.9 & 0.2 \\ 0 & 0 & 0.1 & -0.3 & 0.9 \end{pmatrix} \tag{1.2}$$

From $\mathbf{c} = \mathbf{X}^{-1}\mathbf{q}$, we find

$$\mathbf{c}_3 = \mathbf{X}_3^{-1}\mathbf{q} = \begin{pmatrix} 0.9 & 0.2 & 0 \\ -0.3 & 0.9 & 0.2 \\ 0.1 & -0.3 & 0.9 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.214 \\ 0.9631 \\ 0.3448 \end{pmatrix}$$

$$\mathbf{c}_5 = \mathbf{X}_5^{-1}\mathbf{q} = \begin{pmatrix} 0.9 & 0.2 & 0 & 0 & 0 \\ -0.3 & 0.9 & 0.2 & 0 & 0 \\ 0.1 & -0.3 & 0.9 & 0.2 & 0 \\ 0 & 0.1 & -0.3 & 0.9 & 0.2 \\ 0 & 0 & 0.1 & -0.3 & 0.9 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.0443 \\ -0.1994 \\ 0.9637 \\ 0.3419 \\ 0.0069 \end{pmatrix} \tag{1.3}$$

To estimate how much ISI occurs in each of the above cases, we observe from Fig. 1.1 that to obtain zero ISI (assuming the sample values outside of $t < 3T$ and $t > 3T$ are zero) we need an $\mathbf{X}$ matrix of 7×7. Again by taking Fig. 1.1 and (1.1), we construct such a matrix as below

$$\mathbf{X}_7 = \begin{pmatrix} 0.9 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ -0.3 & 0.9 & 0.2 & 0 & 0 & 0 & 0 \\ 0.1 & -0.3 & 0.9 & 0.2 & 0 & 0 & 0 \\ 0 & 0.1 & -0.3 & 0.9 & 0.2 & 0 & 0 \\ 0 & 0 & 0.1 & -0.3 & 0.9 & 0.2 & 0 \\ 0 & 0 & 0 & 0.1 & -0.3 & 0.9 & 0.2 \\ 0 & 0 & 0 & 0 & 0.1 & -0.3 & 0.9 \end{pmatrix} \tag{1.4}$$

Then multiplying $\mathbf{X}_7$ by and extended version of $\mathbf{c}_3$ or $\mathbf{c}_5$ such that

$$\mathbf{q} = \mathbf{X}\mathbf{c}_{3e} = \begin{pmatrix} 0.9 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ -0.3 & 0.9 & 0.2 & 0 & 0 & 0 & 0 \\ 0.1 & -0.3 & 0.9 & 0.2 & 0 & 0 & 0 \\ 0 & 0.1 & -0.3 & 0.9 & 0.2 & 0 & 0 \\ 0 & 0 & 0.1 & -0.3 & 0.9 & 0.2 & 0 \\ 0 & 0 & 0 & 0.1 & -0.3 & 0.9 & 0.2 \\ 0 & 0 & 0 & 0 & 0.1 & -0.3 & 0.9 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -0.0428 \\ 0 \\ 1 \\ 0 \\ -0.0071 \\ 0.0345 \end{pmatrix} \tag{1.5}$$

As seen from (1.5), we get some residual ISI from sampling instances of $t = -2T,\ 2T,\ 3T$, for the three tap equalize, whereas for the five tap equalizer we will only get the residual ISI at $t = 3T$.

2. (30 Points) In a time varying 4 path channel, the Doppler spread is $B_d = 10^{-3}$ Hz and the following relative mean delays are measured for the individual paths

a) Path1 : 0 second
b) Path2 : 0.7 micro seconds
c) Path3 : 1.1 micro seconds
d) Path4 : 1.56 micro seconds

We wish to transmit *M*ary signals of $M = 16$ over this channel. Calculate the followings

a) Maximum delay, T_m
b) Channel coherence bandwidth, B_{ch}
c) Channel coherence time, T_{ct}

Determine at which bit and symbol durations of the transmitted signal, this channel will assume the following characteristics

a) Overspread channel
b) Underspread channel
c) Slowly fading channel
d) Fast fading channel

Explain what (additional) channel parameters you would measure in order to decide whether this channel is Rayleigh or Ricean type.

Solution : a) From the given path delays, we get

$$T_m = \max\left[\text{delay of path1, delay of path2, delay of path3, delay of path4}\right] = 1.56\ \mu\text{sec} \qquad (2.1)$$

b) $B_{ch} = 1/T_m = 0.64$ MHz

c) $T_{ct} = 1/B_d = 1000$ sec

a), b) According to definitions given in Notes on Multipath channels_Oct 2012_HTE, the channel becomes underspread if $T_m B_d < 1$ and overspread if $T_m B_d > 1$. In this case $T_m B_d = 1.56 \times 10^{-6} \times 10^{-3} = 1.56 \times 10^{-9} \ll 1$, so the channel is undersprаed. This definition seems to be independent of the bit or symbol rate sent through the channel.
c), d) If the symbol rate of our transmission is such that $T_s < T_{ct} < 1000$ sec, then our channel is slowly fading, if on the other hand, the opposite is valid, then we get fast fading channel.

As additional performance measures if $T_s \gg T_m = 1.56\ \mu\text{sec}$, then any fading will be nonfrequency selective, i.e. the whole bandwidth of the transmitted signal will fade simultaneously. But if $T_s \ll T_m = 1.56\ \mu\text{sec}$, then frequency selective fading will occur.

To determine whether this channel will act as Rayleigh or Ricean, we need to have a knowledge of $\alpha(t)$ coefficients, then the coefficient for the direct path (Path1 in this question) is much smaller than the other, we can say that the channel will be Ricean. But if these coefficients have nearly the same value, then the channel will be Rayleigh.

3. (35 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) In an ISI free transmission, we can reach symbol rates of twice the channel bandwidth : True if we state the symbol rate as $R = 1/T$ then the limit of $1/T = 2W$ (the symbol prior to sampling being a sinc function) gives us exactly a rate twice the channel bandwidth.

b) Unlimited channels have no ISI problem : True, since ISI occurs purely because of bandlimited channels. But note that for the unlimited channel not to cause ISI problem, it must have flat frequency response and linear phase response as shown in Notes on ISI_Sept 2012.

c) The probability of error performance of a Rayleigh channel is always better than a Ricean channel : False, since the Ricean channel operates with a strong line of site component whose power is much larger than the reflected components. This was also tested in Matlab.

d) We can transmit at higher bit rates, as the channel becomes underspread : The definition of underspread channel is $T_m B_d < 1$, so this property seems to be independent of the transmission rate.

e) In FSK, message signal waveforms are always orthogonal : Not necessarily, only if we arrange the correlation coefficient to have a value of zero, then message signal waveform are orthogonal to each other.

f) Raised cosine pulse is the best solution to ISI problems : It is better to say one of the optimum and mostly used solutions to ISI problems is the raised cosine pulse.