Çankaya University – ECE Department – ECE 588

Student Name : Duration : 2 hours
Student Number : Open book exam

Questions

1. (35 Points) The attenuation characteristic of a channel is described by the following pdf $f(\alpha) = 10\alpha\exp(-5\alpha^2)$. Identify the type of channel. A signal set of $s_1(t) = 10^{-2} g_T(t)$ and $s_2(t) = -s_1(t)$, where $g_T(t) = \begin{cases} 0.1 & \text{for } 0 < t < 10^{-3} \text{ sec.} \\ 0 & \text{otherwise} \end{cases}$ is transmitted over this channel. Assuming $N_0 = 10^{-10}$ W/Hz, find the probability of error, P_e a) if the channel is only a AWGN channel, where $C(f) = 1$ for $|f| < \infty$ b) if the channel is a AWGN channel plus it has the attenuation characteristics given by the above defined pdf.

Solution: $f(\alpha) = 10\alpha\exp(-5\alpha^2)$ corresponds to Rayleigh channel where $\sigma^2 = 0.1$

E_b (of transmitted signal) $= \int_0^T s_1^2(t)\,dt$

with $T = 10^{-3}$ sec., $s_1(t) = 10^{-3}$ $0 < t < T$

$$E_b = \int_0^{10^{-3}} (10^{-3})^2\,dt = 10^{-6}t\Big|_0^{10^{-3}} = 10^{-9}\ J$$

Thus $SNR = \dfrac{E_b}{N_0}$ with N_0 given as $N_0 = 10^{-10}$ W/Hz

$$SNR = \frac{10^{-9}}{10^{-10}} = 10 \quad \text{or} \quad 10\ dB$$

For binary antipodal PSK (the one given in the question) – Eq. of Proakis 2002

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{2SNR}\right)$$

$$= Q\left(\sqrt{20}\right) =$$

(From Table 4.1 on pp. 152 of Proakis 2002)

On the other hand, from Eq.

of Proakis 2002, P_e in Rayleigh channel

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\rho_e}{1+\rho_e}}\right)$$

where $\rho_e = \frac{E_b}{N_0} E(\alpha^2) = \frac{E_b}{N_0} 2\sigma^2$

Thus P_e (of Rayleigh Ch.) $= 0.5\left(1 - \sqrt{2/3}\right)$

$=$

So it is seen that Rayleigh channel

raises P_e dramatically

2. (35 Points) The OP of a matched filter demodulator, $x(t)$, is shown below. Write for the whole **X** matrix if the FIR filter delay is T and $T/2$. Identify in each case the number of taps. Find the tap coefficients c_{-1}, c_0 and c_1 if a three tap equalizer is used. Draw the block diagram of the equalizer.

For the waveform see Q1 of EEM 468 MT2 (6.05.2008) Q1, sample values are also given here

	$t=-3T$	$t=-2.5T$	$t=-2T$	$t=-1.5T$
$x(t) =$	0,	0.18,	0.1,	−0.18

$t=-T$	$t=-0.5T$	$t=0$	$t=+0.5T$
0.2,	0.6,	0.9	0.63

$t=T$	$t=1.5T$	$t=2T$	$t=2.5T$	$t=3T$
−0.27,	−0.12,	0.1,	0.07,	0

For a complete solution see enclosed sheets and also EEM 468 MT2, Q

3. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) If $T_m = 1$ msec. and $B_d = 5$ Hz , then channel is overspread : False

Since $T_m B_d = 10^{-3} \times 5 Hz \ll 1$, so the channel is underspread

b) The tap delay line model of the channel represents the arrival of the same transmitted signal via different paths : True

Since in this model, the channel is shown as series of filters giving delays of $1/W$ with a total length of T_m such that number of filters $(L) = T_m W$

c) If bandwidth of transmitted signal is more than $1/T_m$, the channel acts as a single path channel : False

If $W > \frac{1}{T_m}$ or $T_m W > 1$, then $L > 1$

so the channel has more than one (resolvable) path

d) To prevent ISI, we use raised cosine pulses instead of rectangular pulses : True

The use of raised cosine pulses prevents ISI

e) An MMSE equalizer takes into account noise characteristics as well as the channel response : True. Equalization based purely on $G_E(f) = 1/C(f)$ caters for signal attenuation (and perhaps phase correction), but MMSE equalizer takes into account noise characteristics as well as signal attenuation so that

f) Duobinary and modified duobinary signals remove ISI partially : True, since

in Duobinary $x(nT) = \begin{cases} 1 & \text{for } n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$

in modified Duobinary $x(nT) = \begin{cases} 1 & n = \mp 1 \\ 0 & \text{otherwise} \end{cases}$

excessive signal amplification can be avoided.