Çankaya University – ECE Department – ECE 588

Student Name :
Student Number :

Duration : 2 hours
Open book exam

Questions

1. (35 Points) The attenuation characteristics of a channel is described by the following pdf $f(\alpha)=0.1\delta(\alpha)+0.2\delta(\alpha-1)+0.15\delta(\alpha-2)$. A signal set of $s_1(t)=g_T(t)$ and $s_2(t)=-s_1(t)$, where $g_T(t)=\begin{cases}12\text{ mV} & \text{for } 0<t<10^{-4}\text{ sec.}\\ 0 & \text{otherwise}\end{cases}$ is transmitted over this channel. Assuming $N_0=10^{-10}$ W/Hz, find the probability of error, P_e a) for each attenuation component b) Overall P_e covering all attenuation components. For these calculations, you may use equations listed on page 687 of Proakis, Communication Systems Engineering (2002) book.
Identify how many paths there are in this channel. Assuming that there is a time delay of 1 μsecond between the nearest paths, estimate the multipath spread (T_m) of this channel, draw a model of this channel in terms of tapped delay lines. If Doppler spread $B_d=100$ Hz, state whether this channel is overspread or underspread.

Solution: For a fixed value of α

$$P_e(\alpha)=\sqrt{\frac{2\alpha^2\mathcal{E}_b}{N_0}}, \quad \mathcal{E}_b=\int_0^{10^{-4}}(12\times10^{-3})^2\,dt$$

$= 144\times10^{-10}$ Joule $= 14.4$ n Joule

α has three values at $\alpha=0$, $\alpha=1$, $\alpha=2$ with respective probabilities of 0.1, 0.2, 0.15 (this means a total of 0.45, hence the remaining 0.55 is distributed to other attenuation coefficients, which we disregard in this question)

According to Eq (10.1.24) of Proakis 2002 on pp. 687

$$P_{e_{\alpha=0}} = Q\left(\sqrt{\frac{2\alpha^2 \mathcal{E}_b}{N_0}}\right) = Q(0) = 0.5$$

$$P_{e_{\alpha=1}} = Q\left(\sqrt{\frac{2\times 144\times 10^{-10}}{10^{-10}}}\right) = Q(\sim 17) = 4.1062\times 10^{-65}$$
$$= \sim 0$$

Note that in Matlab $erfc(x)$ is defined, where

$$Q(x) = 0.5\, erfc(x/\sqrt{2})$$

$$P_{e_{\alpha=2}} = Q\left(\sqrt{\frac{2\times 4\times 144\times 10^{-10}}{10^{-10}}}\right) = Q(34)$$

$$= 1.11\times 10^{-253} \rightarrow 0$$

According to given pdf $f(\alpha)$, there are three paths in this channel with

$\alpha = 0$, $\alpha = 1$, $\alpha = 2$

$T_{m_0} = 0$ $T_{m_1} = 1\mu s$ $T_{m_2} = 2\mu s$

Multipath spread $T_m = 2\ \mu sec$

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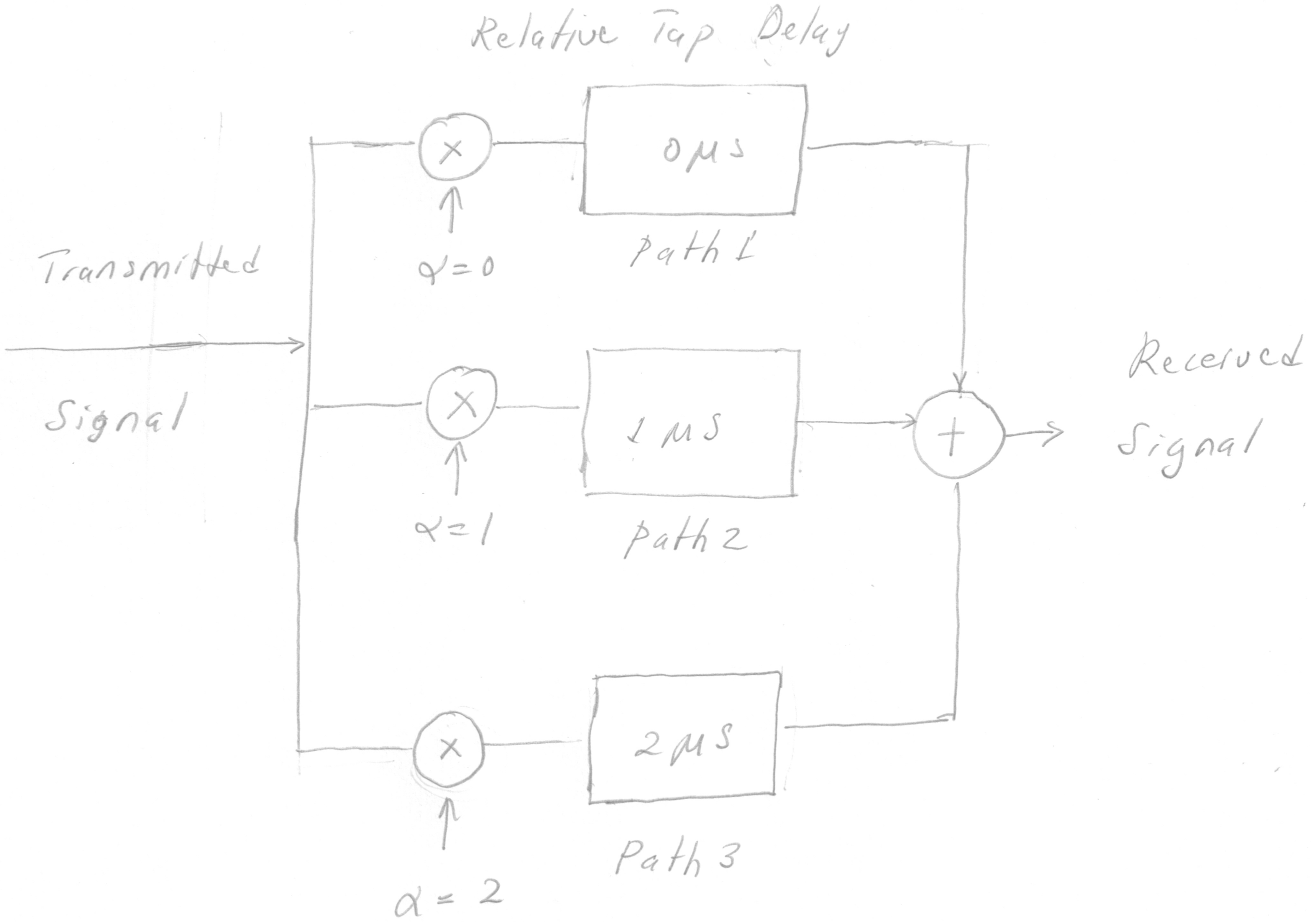
Since T_s (symbol duration or bit duration) = 100 μs

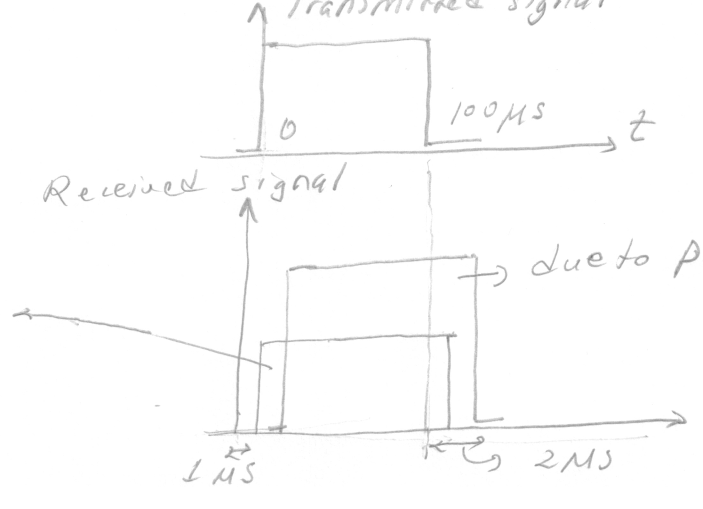
and W (Bandwidth of the signal) $\approx \frac{1}{T_s} = 10$ kHz

the three multipath components are not resolvable and

the channel can be represented as



The below waveforms also illustrate transmitted and received signals



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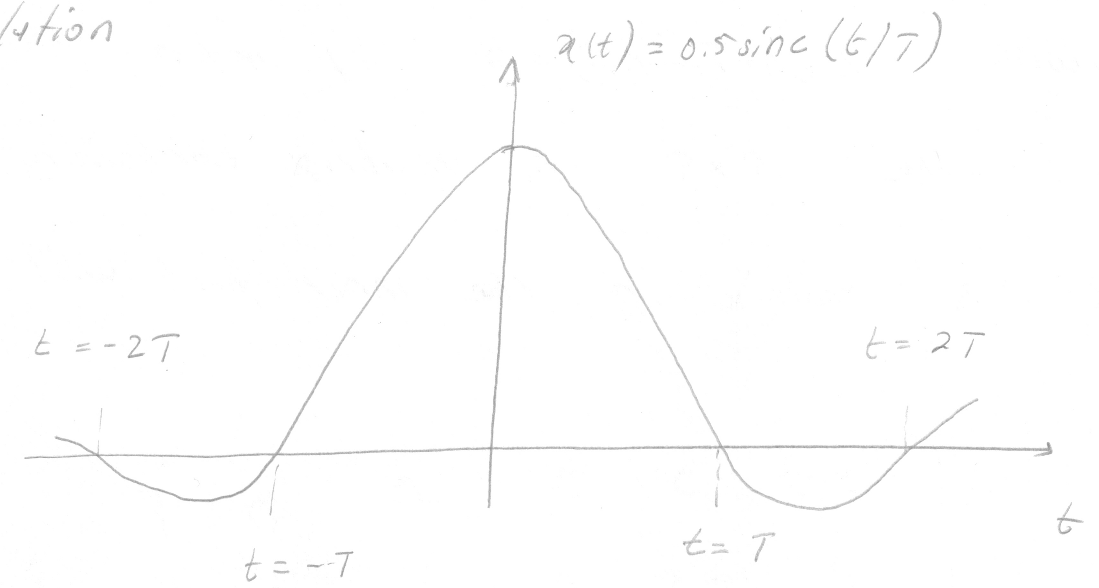
To determine wether the channel in question is underspread or overspread, we have to calculate the product $T_m B_d = 2\mu s \times 100 Hz = 2\times10^{-4} \ll 1$ so the channel is underspread.

2. (35 Points) The OP of a matched filter demodulator, $x(t) = 0.5\text{sinc}(t/T)$. Write the 5 x 5 and 3 x 3 **X** matrix if the FIR filter delay is T and $T/2$ (in total four matrices). Find the tap coefficients of the equalizer for each of the four cases.

Solution



To find the elements of X matrix of 5x5 and 3x3 at T and T/2 FIR delays, we have to calculate $x(mT - nT)$ and $x(mT - nT/2)$

for $m = -2, -1, 0, +1, +2$ and

$n = -2, -1, 0, +1, +2$

For $x(mT - nT)$ - FIR delays of T

$m = -2,\ n = -2,\quad x(t = 0) = 0.5$

$m = -2,\ n = -1,\quad x(t = -T) = 0$

$m = -2,\ n = 0,\quad x(t = -2T) = 0$

$m = -2,\ n = 1,\quad x(t = -3T) = 0$

$m = -2,\ n = 2,\quad x(t = -4T) = 0$

According to above, at other m, n values

for $m = n$ $\quad x(mT - nT) = 0.5$ if $m = n$

otherwise $\quad x(mT - nT) = 0$ if $m \neq n$

Thus the 5×5 X matrix containing 3×3 X matrix in the inner part will be

$$X_{55}^{T} = \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 \\ 0 & \boxed{\begin{matrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{matrix}} & & & 0 \\ 0 & & & & 0 \\ 0 & & & & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \rightarrow X_{33}^{T}$$

$= 0.5 \, I_{55}$

5×5 identity matrix

The equalizer coefficient in this case will be given by

condition for zero ISI

$$C_{55}^{T} = inv(X_{55}^{T})\, q = inv(X_{55}^{T}) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$C_{33}^{T} = inv(X_{33}^{T})\, q = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

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For the case of $x(mT - nT/2) \rightarrow$ FIR delays of $T/2$

$m=-2,\ n=-2,\ x(t=-T)=0$

$m=-2,\ n=-1,\ x(t=-1.5T)=-0.1061$

$m=-2,\ n=0,\ x(t=-2T)=0$

$m=-2,\ n=1,\ x(t=-2.5T)=0.0637$

$m=-2,\ n=2,\ x(t=-3T)=0$

$m=-1,\ n=-2,\ x(t=0)=0.5$

$m=-1,\ n=-1,\ x(t=-0.5T)=0.3138$

$m=-1,\ n=0,\ x(t=-T)=0$

$m=-1,\ n=1,\ x(t=-1.5T)=-0.1061$

$m=-1,\ n=2,\ x(t=-2T)=0$

$m=0,\ n=-2,\ x(t=T)=0$

$m=0,\ n=-1,\ x(t=0.5T)=0.3138$

$m=0,\ n=0,\ x(t=0)=0.5$

$m=0,\ n=1,\ x(t=-0.5T)=0.3138$

$m=0,\ n=2,\ x(t=-T)=0$

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$m=1, \quad n=-2, \quad x(t=2T)=0$

$m=1, \quad n=-1, \quad x(t=1.5T)=-0.1061$

$m=1, \quad n=0, \quad x(t=T)=0$

$m=1, \quad n=+1, \quad x(t=0.5T)=0.3138$

$m=2, \quad n=2, \quad x(t=0)=0.5$

$m=2, \quad n=-2, \quad x(t=3T)=0$

$m=2, \quad n=-1, \quad x(t=2.5T)=0.0637$

$m=2, \quad n=0, \quad x(t=2T)=0$

$m=2, \quad n=1, \quad x(t=1.5T)=-0.1061$

$m=2, \quad n=2, \quad x(t=T)=0$

The $X_{55}^{T/2}$ and inner $X_{33}^{T/2}$ matrices are shown overleaf

ECE 588 Final Exam - 11.01.2011 - Q.2 - Insert (3)

$$X_{55}^{T/2} = \begin{bmatrix} 0 & -0.1061 & 0 & -0.0637 & 0 \\ 0.5 & 0.3138 & 0 & -0.1061 & 0 \\ 0 & 0.3138 & 0.5 & 0.3138 & 0 \\ 0 & -0.1061 & 0 & 0.3138 & 0.5 \\ 0 & 0.0637 & 0 & -0.1061 & 0 \end{bmatrix}$$

$X_{33}^{T/2}$

$$C_{55}^{T/2} = inv\left(X_{55}^{T/2}\right) q = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \quad C_{33}^{T/2} = inv\left(X_{33}^{T/2}\right) q = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

Interpretations of above results are that sinc pulse is already ISI free pulse shape including Transmitter Filter, Channel Response and Receive Filter. Thus coefficients of c matrix come out to be zero for $n \neq 0$, i.e zero ISI condition is acheived even before equalization is applied.

3. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) ISI can be reduced by transmitting a signal that has a bandwidth smaller than the channel bandwidth : True, in any case we cannot eliminate ISI if bandwidth of transmitted signal is greater than channel bandwidth

b) There are more paths in a Rayleigh channel than a Ricean one: True, since in a Rayleigh channel, the number of path approaches infinity

c) An unlimited channel is always better than bandlimited channel : True, an unlimited channel is the most desirable case from the view point that the transmitted signal will experience no distortion

d) PSK uses more bandwidth than ASK : True, since PSK slices the duration of one symbol into two intervals

e) ISI can be eliminated if the product of transmitter filter (frequency) response, channel response and receiver filter response is equated to raised cosine function : True

This means $G_T(f)\,C(f)\,G_R(f) = X_{rc}(f)$

f) Multipath phenomena occurs due to moving objects between transmitter and receiver :

False, multipath situation exist even when there are moving objects around, Doppler shift occurs when moving objects are involved.