Çankaya University – ECE Department – ECE 588 (MT)

Student Name : Date : 06.04.2015
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Questions

1. (70 Points) The time waveforms of the signal set, $s_1(t)$ and $s_2(t)$ are given in Fig. 1.1.

   a. Identify the type of modulation and dimensionality in this signal set. Write mathematical expression for $s_1(t)$ and $s_2(t)$ and the corresponding basis functions, $\psi_1(t)\ \cdots\ \psi_N(t)$ and plot $\psi_1(t)\ \cdots\ \psi_N(t)$. Write for the signal vectors $\mathbf{s}_1$ and $\mathbf{s}_2$, and plot the corresponding constellation diagram. Find the distance between signal vector ends.
   b. Draw the demodulator as correlator and matched filter. Assuming that the signal $s_1(t)$ from constellation is transmitted, find the outputs of the correlator and matched filter.
   c. Find the probability of error and decision regions via the evaluations of correlation metrics $C(\mathbf{r}, \mathbf{s}_m)$ again assuming $s_1(t)$ was transmitted and the probability of sending $s_1(t)$ or $s_2(t)$ is unequal.

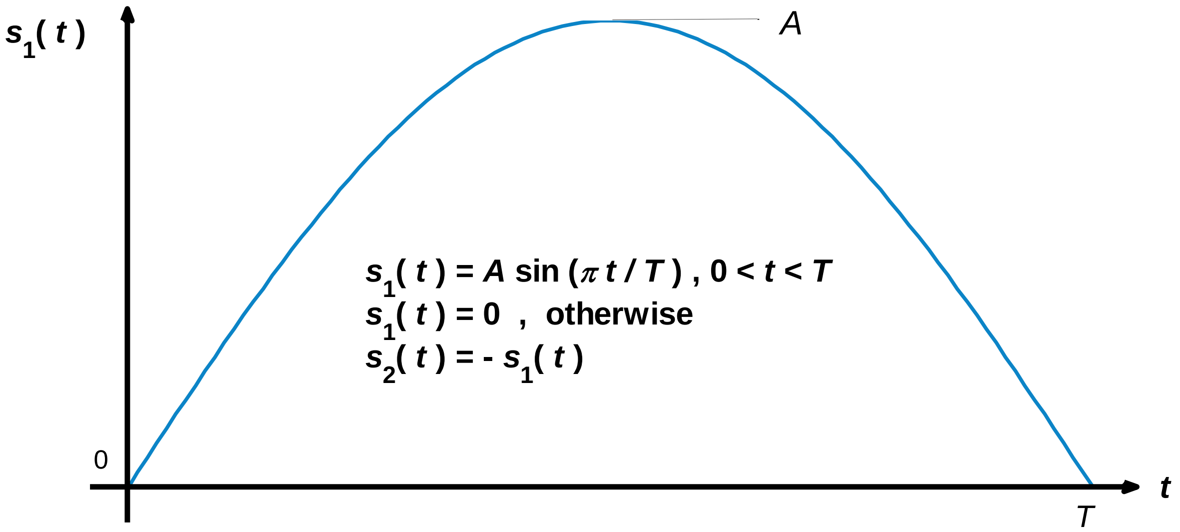


Fig. 1.1 The time waveforms, $s_1(t)$ and $s_2(t)$ for Q1.

Solution : a. From Fig. 1.1, we see that $s_1(t)$ and $s_2(t)$ will be

$$s_1(t)=\begin{cases}A\sin(\pi t/T) & 0\le t\le T\\ 0 & \text{otherwise}\end{cases}\quad,\quad s_2(t)=\begin{cases}-A\sin(\pi t/T) & 0\le t\le T\\ 0 & \text{otherwise}\end{cases}\qquad(1.1)$$

The energies of these signals can be found from

$$\varepsilon_{s_1} = \int_{-\infty}^{\infty} s_1^2(t)\, dt = \int_0^T s_1^2(t)\, dt = A^2 \int_0^T \sin^2(\pi t / T)\, dt = \frac{A^2 T}{2} = \varepsilon_{s_2} = \varepsilon_s \qquad (1.2)$$

Both signals can be represented by single orthonormal basis function. Hence, $s_1(t)$ and $s_2(t)$ will constitute binary ASK, where $M = 2,\ N = 1$. The related basis function, $\psi(t)$ will have similar appearance to that of Fig. 1.1 and can be written as

$$\psi(t) = \begin{cases} \sqrt{2/T}\sin(\pi t / T) & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases} \qquad (1.3)$$

$s_1(t)$ and $s_2(t)$ in terms of $\psi(t)$ and the corresponding vectorial representations are given below

$$s_1(t) = A\sqrt{T/2}\psi(t)\ ,\ s_2(t) = -A\sqrt{T/2}\psi(t)$$
$$\mathbf{s}_1 = \left[A\sqrt{T/2}\right]\ ,\ \mathbf{s}_2 = \left[-A\sqrt{T/2}\right]\ ,\ d_{12} = 2A\sqrt{T/2} \qquad (1.4)$$

The constellation diagram is given in Fig. 1.2.

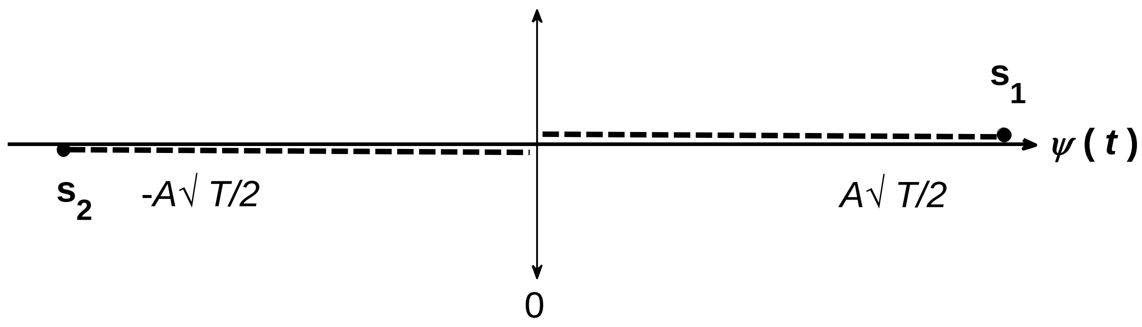
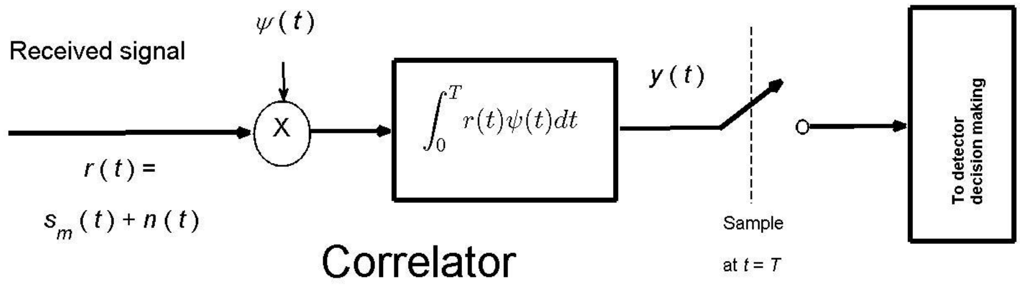


Fig. 1.2 Constellation diagram for $s_1(t)$ and $s_2(t)$ of Fig. 1.1.

b. The demodulator as correlator and matched filter is shown in Fig. 1.3.

The output from the demodulator, when $s_1(t)$ was transmitted



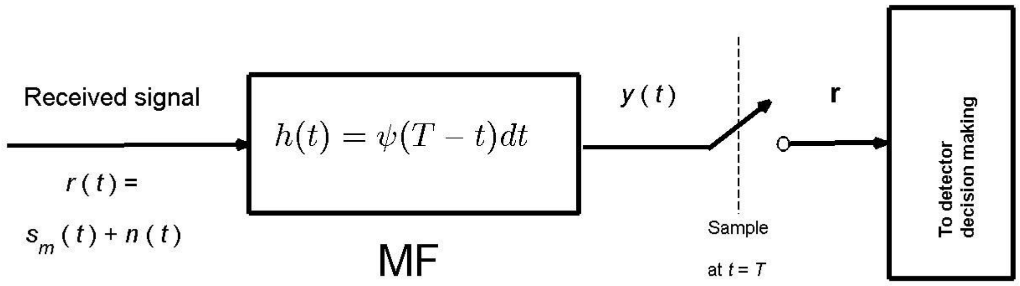


Fig. 1.3 Demodulator for $s_1(t)$ and $s_2(t)$ as correlator and MF.

Correlator

$$\mathbf{r} = y(t=T) = \int_0^T r(t)\ \psi(t)\ dt = \int_0^T \left[s_1(t) + n(t) \right]\ \psi(t)\ dt$$

$$= A\sqrt{T/2} + n_b \quad , \quad n_b = \int_0^T n(t)\ \psi(t) dt$$

MF

$$\mathbf{r} = y(t=T) = \int_0^T r(\tau)\ \psi(\tau)\ d\tau = A\sqrt{T/2} + n_b = \varepsilon_s^{0.5} + n_b \qquad (1.5)$$

c. The evaluation correlation metrics is given below

We start with (6.10) of Notes on Dimensionality of Signals_Sept 2012_HTE, which is

$$\text{Max}\left[P\left(\mathbf{s}_m | \text{r}\right) \right] \equiv \text{Max}\left[f\left(\mathbf{r}\ |\mathbf{s}_m\right) P\left(\mathbf{s}_m\right) \right] \qquad (1.6)$$

Using (5.8) and (6.10) of the same notes, we get

$$\text{Max}\left[P\left(\mathbf{s}_m | \text{r}\right) \right] \equiv \text{Max}\left\{ \frac{-1}{2} \ln\left(\pi N_0\right) - \frac{1}{N_0} \sum_{n=1}^{N} \left(r_n\ - s_{mn}\right)^2 + \ln\left[P\left(\mathbf{s}_m\right) \right] \right\} \qquad (1.7)$$

Converting (1.7) into correlation metrics, we obtain

$$
\begin{aligned}
C(\mathbf{r},\mathbf{s}_m) &= \frac{-1}{2}\ln(\pi N_0) + N_0 \ln\left[P(\mathbf{s}_m)\right] - \sum_{n=1}^{N}(r_n - s_{mn})^2 \\
&= N_0 \ln\left[P(\mathbf{s}_m)\right] + 2\,\mathbf{r}\cdot\mathbf{s}_m - \overbrace{\frac{1}{2}\ln(\pi N_0) - \|\mathbf{s}_m\|^2}^{\text{common terms}} \\
&\equiv N_0 \ln\left[P(\mathbf{s}_m)\right] + 2\,\mathbf{r}\cdot\mathbf{s}_m
\end{aligned}
\tag{1.8}
$$

Now evaluating $C(\mathbf{r},\mathbf{s}_m)$ for $m=1,\ 2$

$$
\begin{aligned}
&m=1\ ,\ C(\mathbf{r},\mathbf{s}_1) = N_0 \ln\left[P(\mathbf{s}_1)\right] + 2\,\mathbf{s}_1\cdot\mathbf{r} = N_0\ln(p) + 2A\sqrt{T/2}\left(A\sqrt{T/2}+n_b\right) \\
&m=2\ ,\ C(\mathbf{r},\mathbf{s}_2) = N_0 \ln\left[P(\mathbf{s}_2)\right] + 2\,\mathbf{s}_2\cdot\mathbf{r} = N_0\ln(1-p) - 2A\sqrt{T/2}\left(A\sqrt{T/2}+n_b\right)
\end{aligned}
\tag{1.9}
$$

For correct decision, it should be

$$
C(\mathbf{r},\mathbf{s}_1) > C(\mathbf{r},\mathbf{s}_2) \tag{1.10}
$$

Substituting in (1.10) from (1.9), we get

$$
\begin{aligned}
&N_0\ln(p) + 2A\sqrt{T/2}\left(A\sqrt{T/2}+n_b\right) > N_0\ln(1-p) - 2A\sqrt{T/2}\left(A\sqrt{T/2}+n_b\right) \\
&4\varepsilon_s^{0.5}\overbrace{\left(\varepsilon_s^{0.5}+n_b\right)}^{\mathbf{r}} > N_0\ln\frac{1-p}{p} \quad\rightarrow\quad \mathbf{r} > \frac{N_0}{4\varepsilon_s^{0.5}}\ln\frac{1-p}{p}
\end{aligned}
\tag{1.11}
$$

Which is the same result as given in (6.18) of Notes on Dimensionality of Signals_Sept 2012_HTE.

For given operating conditions, $\frac{N_0}{4\varepsilon_s^{0.5}}$ will be fixed, hence (1.11) basically becomes

$$
\mathbf{r} > \frac{N_0}{4\varepsilon_s^{0.5}}\ln\frac{1-p}{p} \quad\rightarrow\quad \mathbf{r} > \ln\frac{1-p}{p}\bigg|_{\frac{N_0}{4\varepsilon_s^{0.5}}\rightarrow\text{constant}} \tag{1.12}
$$

Graphically, (1.12) is depicted in Fig. 1.4. Note that $p = P(\mathbf{s}_1)$ and $1-p = P(\mathbf{s}_1)$.

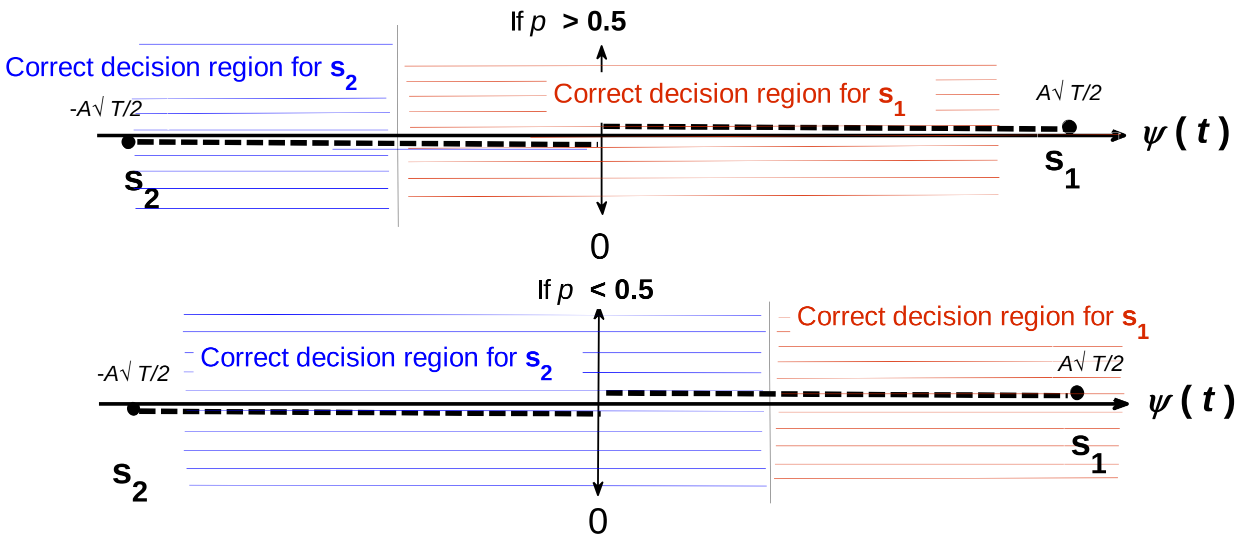


Fig. 1.4 Variations of decision regions with p .

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) The role of the matched filter is to scale the input signal : False, matched filter is a correlator, it gives the maximum output, when there is 100 % correlation between the input and its time response.

b) Under the conditions of equal energy, ASK has higher probability of error that PSK, QAM and FSK : True, ASK has the worst error performance amoung the modulation types, since it uses only one dimension of the signal space.

c) In PSK, symbols are transmitted as $s_1(t)\ \cdots\ s_M(t)$ in a cyclic manner : Any modulator transmits the symbols according to a sequence determined by the message symbol, i.e. in a random manner.

d) Optimum detector uses the correlation metrics to make a decision : True, when it uses the results of correlation metrics, it decides for the $\mathbf{s}_m$, that maximizes the correlation metrics value.

e) ISI occurs, since the transmitted signal has infinite bandwidth : Not exactly, ISI occurs if the spectral view of the message signal is modified when passing through the channel.

f) An unlimited channel introduces no ISI : True, an unlimited channel cannot introduce ISI, but if it has no frequency dependence.